

Model (2016-2017)

Subiectul I

1. $10 + (3+7) \cdot 10 = 10 + 10 \cdot 10 = 10 + 100 = 110$

2. $18 : 6 = 3$ li/caiet
 $3 \times 3 = 9$ li (3 caiete)

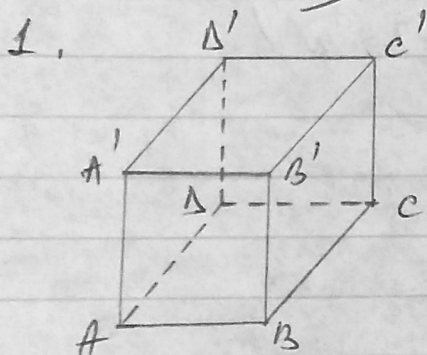
3. 99

4. $m(\angle ABC) = 60^\circ$

5. Suma tuturor muchilor tetraedrului = $6 \cdot 5 = 30$ cm

6. Nr. elevilor care au optat pt filiera teoretică = $\frac{30}{2} = 15$ elevi
sau $30 \cdot \frac{50}{100} = 15$ elevi

Subiectul II



2. $a = 3^{100} : 3^{98} = 3^{100-98} = 3^2 = 9$

$b = 3 \cdot 2 - 2 = 6 - 2 = 4 = 2^2$

$m_g = \sqrt{a \cdot b} = \sqrt{3^2 \cdot 2^2} = 3 \cdot 2 = 6$

3. $\frac{x}{5} = \frac{y}{4} = \frac{x+y}{5+4} = \frac{54}{9}$

$x+y=54$

$\frac{x}{5} = \frac{54}{9} \Rightarrow x = \frac{54 \cdot 5}{9} = 6 \cdot 5 = 30$

$\frac{y}{4} = \frac{54}{9} \Rightarrow y = \frac{4 \cdot 54}{9} = 4 \cdot 6 = 24$

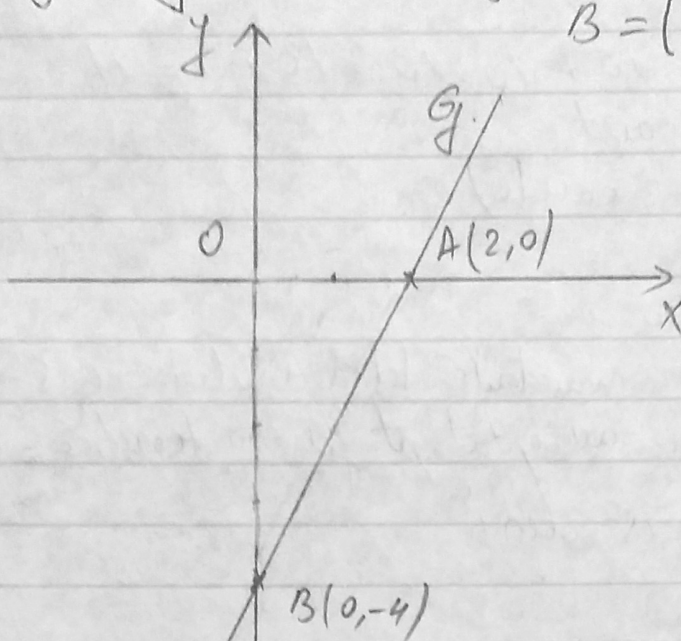
$$4. f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x - 4$$

$$a) G_f \cap O_x \Rightarrow y = 0 \Leftrightarrow f(x) = 0 \Rightarrow 2x - 4 = 0 \Rightarrow x = \frac{4}{2} = 2$$

$$A = (2, 0)$$

$$G_f \cap O_y \Rightarrow x = 0 \Leftrightarrow f(0) = y \Rightarrow 2 \cdot 0 - 4 = y \Rightarrow y = -4$$

$$B = (0, -4)$$



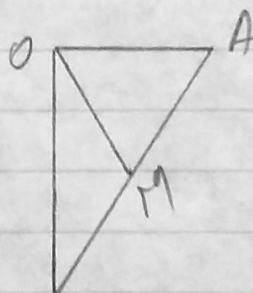
b) $\triangle AOB$ - dreptunghiic ($m(\hat{O}) = 90^\circ$)

$$OA = 2$$

$$OB = 4$$

M mijlocul lui AB.

$$OM = \frac{AB}{2}$$



Aplicăm th. Pitagora:

$$AB^2 = OB^2 + OA^2 = 2^2 + 4^2 = 4 + 16 = 20$$

$$AB = \sqrt{20} = 2\sqrt{5}$$

$$OM = \frac{2\sqrt{5}}{2} = \sqrt{5}$$

$$5. E(x) = \frac{(x-2)^2 - 2(x-2) + 1}{x^2 - 9} \cdot \frac{x+3}{x-3} =$$

$$= \frac{[(x-2) - 1]^2}{(x-3)(x+3)} \cdot \frac{x+3}{x-3} = \frac{(x-2-1)^2}{(x-3)(x-3)} =$$

$$= \frac{(x-3)^2}{(x-3)^2} = 1$$

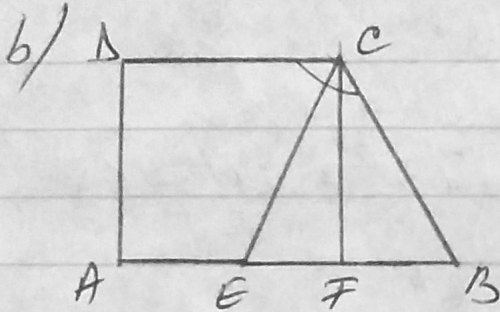
Am aplicat:

$$a^2 - 2a + 1 = (a-1)^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

Subiectul III

$$1. a) A_{ABCD} = \frac{h(B+b)}{2} = \frac{AD(AB+DC)}{2} = \frac{20\sqrt{3}(100+60)}{2} = 20\sqrt{3} \cdot 160 = 3200\sqrt{3} \text{ m}^2$$



Tragem din $C \perp AB \Rightarrow$

$$\Rightarrow CF = AD = 20\sqrt{3} \text{ m}$$

$$DC \parallel AF \Rightarrow DC = AF = 60 \text{ m} \Rightarrow$$

$$AB = 100 \text{ m}$$

$$\Rightarrow FB = 100 - 60 = 40 \text{ m}$$

$$\Delta CFB - \text{dreptunghi} \Rightarrow BC^2 = FC^2 + FB^2 = (20\sqrt{3})^2 + 40^2 = 40^2 \cdot 3 + 40^2 = 40^2(3+1) = 40^2 \cdot 4$$

$$BC = \sqrt{40^2 \cdot 4} = 40 \cdot 2 = 80 \text{ m}$$

$$\sin(\widehat{FCB}) = \frac{FB}{BC} = \frac{40}{80} = \frac{1}{2} \Rightarrow m(\widehat{FCB}) = 30^\circ$$

$$m(\widehat{BCD}) = m(\widehat{BCF}) + m(\widehat{FCD}) = 30^\circ + 90^\circ = 120^\circ$$

$$c) A_{ABCD} = A_{AECD} + A_{BEC} \Rightarrow A_{ABCD} = 2A_{BEC}$$

$$A_{AECD} = A_{BEC} \Rightarrow A_{BEC} = \frac{CF \cdot EB}{2} \Rightarrow$$

$$\Rightarrow \frac{2 \cdot 20\sqrt{3} \cdot EB}{2} = 3200\sqrt{3} \Rightarrow 40\sqrt{3} \cdot EB = 3200\sqrt{3} \Rightarrow$$

$$\Rightarrow EB = \frac{3200\sqrt{3}}{40\sqrt{3}} = 80 \text{ m}$$

$$EB = BC = 80 \text{ m}$$

$$\Delta BEC, \sin(\widehat{BCE}) = \frac{BE}{BC} = \frac{80}{80} = 1 \Rightarrow m(\widehat{BCE}) = 90^\circ \Rightarrow$$

$\Rightarrow \Delta CEB - \text{echilateral}$

$$2. a) A_{\text{trajei}} = \pi \cdot R^2 = \pi \cdot OA^2 = \pi \cdot 3^2 = 9\pi \text{ cm}^2$$

$$R = OA$$

$$b) A_l = \pi \cdot R \cdot G$$

$$R = OA, G = VA = VB$$

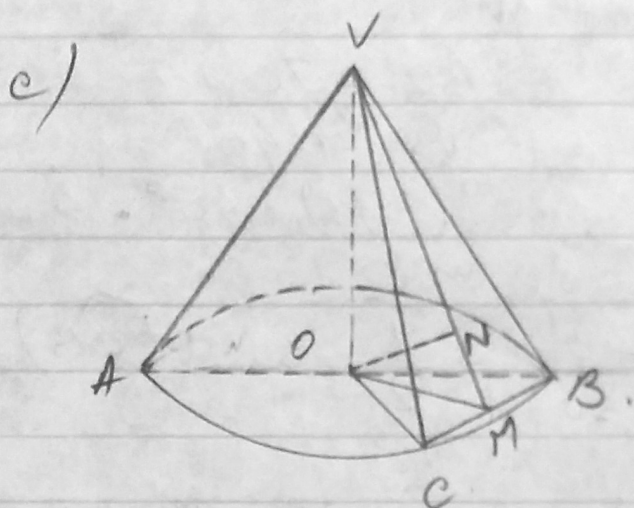
$$VO = h \text{ conului}$$

$$\Delta VOA \text{ - dreptunghiic (m}(\hat{O}) = 90^\circ)$$

$$\text{Teorema Pitagora } VA^2 = AO^2 + VO^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$VA = \sqrt{25} = 5 \text{ cm}$$

$$A_l = \pi \cdot 3 \cdot 5 = 15\pi \text{ cm}^2$$



$$d(O, (VBC)) = ON$$

$$\begin{array}{l} \text{Ducem } ON \perp (VBC) \\ N \in (VBC) \\ BC \subset (VBC) \end{array} \Bigg| \Rightarrow ON \perp BC$$

$$\begin{array}{l} VO \perp OB \\ OB \subset (BOC) \end{array} \Bigg| \Rightarrow VO \perp (BOC) \Bigg| \Rightarrow VO \perp BC$$

$$\begin{array}{l} ON \cap VO = \{O\} \end{array} \Bigg| \Rightarrow$$

$$\Rightarrow BC \perp (VON) \Rightarrow BC \perp VN$$

$$\begin{array}{l} VN \cap BC = \{M\} \\ VB = VC \end{array} \Bigg| \Rightarrow M \text{ este mijlocul } BC$$

$$\Rightarrow VM \perp BC$$

$$\Delta VMN \text{ dreptunghiic} \Rightarrow VM^2 = VB^2 - MB^2$$

$$\Delta BOC \text{ dreptunghiic (ipotenuza)} \Rightarrow BC^2 = OB^2 + OC^2 = 9 + 9 = 18 \Rightarrow$$

$$BC = \sqrt{18} = 3\sqrt{2} \text{ cm}$$

$$\Rightarrow VM^2 = 5^2 - \left(\frac{3\sqrt{2}}{2}\right)^2 = 25 - \frac{9 \cdot 2}{4} = \frac{25 \cdot 4 - 18}{4} = \frac{100 - 18}{4} = \frac{82}{4}$$

$$VM = \sqrt{\frac{82}{4}} \text{ cm}$$

$$\Delta BOC - \text{dreptunghi} \text{ isoscel } (OB=OC=R=3\text{cm}) \Rightarrow$$

$$\Rightarrow OM = \frac{BC}{2} \Rightarrow OM = \frac{3\sqrt{2}}{2}$$

$$\text{Cum } VO \perp OB \quad \left| \Rightarrow VO \perp (COB) \right. \\ \left. VO \perp OC \quad \left| \Rightarrow VO \perp OM \Rightarrow \right. \right.$$

$\Rightarrow \Delta VOM$ - dreptunghi

$$\text{Cum } ON \perp (VOB) \quad \left| \Rightarrow ON \perp VM \Rightarrow \right. \\ \left. VM \subset (VOB) \right.$$

$\Rightarrow ON$ înălțime în ΔVOM

Area ΔVOM este: $\frac{ON \cdot VM}{2}$ sau $\frac{VO \cdot OM}{2} \Rightarrow$

$$\Rightarrow \frac{ON \cdot VM}{2} = \frac{VO \cdot OM}{2} \quad \left(\begin{array}{l} \times 2 \\ \Rightarrow \end{array} \right) ON \cdot VM = VO \cdot OM \Rightarrow$$

$$\Rightarrow ON = \frac{VO \cdot OM}{VM} \Rightarrow ON = \frac{6 \cdot \frac{3\sqrt{2}}{2}}{\frac{\sqrt{82}}{2}} = 6\sqrt{2} \cdot \frac{2}{\sqrt{82}}$$

$$\Rightarrow ON = \frac{12\sqrt{2}}{\sqrt{2} \cdot \sqrt{41}} = \frac{12}{\sqrt{41}} \Rightarrow ON = \frac{12\sqrt{41}}{41}$$