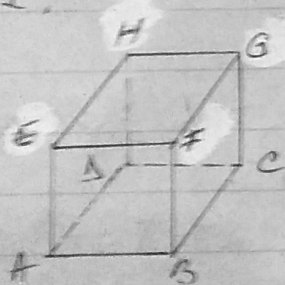


Varianta 6 (2016-2017)

- I.
- $20 - 20 : 2 = 20 - 10 = 10$
 - $30 : 6 = 5$ lei un caiet
 $5 \times 3 = 15$ lei
sau $30 : 2 = 15$ lei (deoarece 3 este jumătatea lui 6)
 - $A = \{1, 2, 3, 4\}$
 $B = \{4, 6, 8\}$
 $A \cap B = \{4\}$
 - $l = 6 \text{ cm} \Rightarrow A_{\square} = l^2 = 6^2 = 36 \text{ cm}^2$
 - Suma lungimilor tuturor tetraedrului = 12 cm
Tetraedrul are 6 laturi (muchii)
 $AB = 12 : 6 = 2 \text{ cm}$
 - Nr. total al elevilor cls. a viii a = $30 + 28 = 58$ elevi

II. 1.



$$2. (1 + 0,5)(1 - 0,5) + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{5}{4}$$

$$(1 + 0,5)(1 - 0,5) + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - (0,5)^2 + \frac{1^2}{(\sqrt{2})^2} = 1 - 0,25 + \frac{1}{2} =$$

$$(a+b)(a-b) = a^2 - b^2, \quad a=1, \quad b=0,5$$

$$= 1 - \frac{25}{100} + \frac{1}{2} = 1 - \frac{1}{4} + \frac{1}{2} = \frac{4-1+2}{4} = \frac{5}{4}$$

$$3. a = ?, b = ? , \boxed{m_a = \frac{a+b}{2}}$$

$$m_a = 150$$

$$\frac{a}{b} = \frac{1}{2} \Rightarrow b = 2a$$

$$m_a = \frac{a+b}{2} \Rightarrow \frac{a+b}{2} = 150$$

$$\Rightarrow \frac{a+2a}{2} = 150 \Leftrightarrow$$

$$\Leftrightarrow 3a = 300 \Rightarrow a = 300 : 3 \Rightarrow a = 100$$

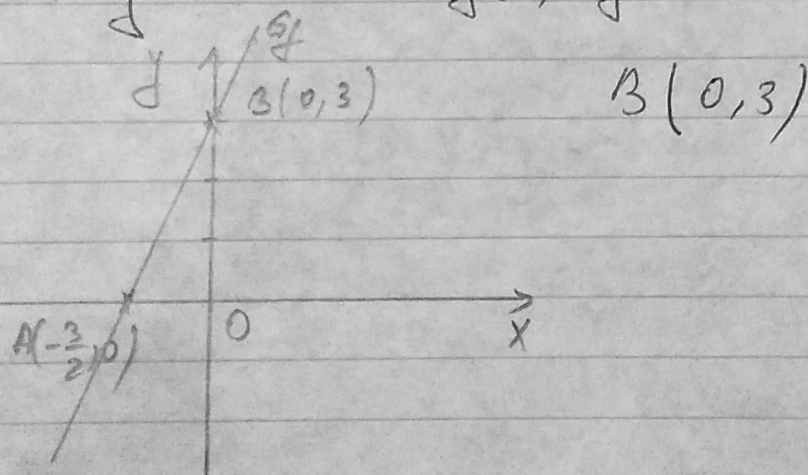
$$b = 2 \cdot 100 = 200$$

$$4. f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 3$$

$$a) G_f \cap O_x \Rightarrow y = 0 \Leftrightarrow f(x) = 0 \Rightarrow 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$

$$A\left(-\frac{3}{2}, 0\right)$$

$$G_f \cap O_y \Rightarrow x = 0 \Leftrightarrow f(0) = y \Rightarrow 2 \cdot 0 + 3 = y \Rightarrow y = 3$$



$$b) \text{abscisa} = \text{ordonată} \Rightarrow x = y \mid \rightarrow f(x) = x \Leftrightarrow 2x + 3 = x \Leftrightarrow$$

$$y = f(x) \mid \Leftrightarrow 2x - x = -3 \Rightarrow x = -3$$

Deci, punctul în care abscisa = ordonată este $A(-3, 3)$

$$5. E(x) = \frac{(x+2)^2 - 9}{x^2 - 25} \cdot \frac{x-1}{x-5} = \frac{(x+2-3)(x+2+3)}{(x-5)(x+5)} \cdot \frac{x-5}{x-1} =$$

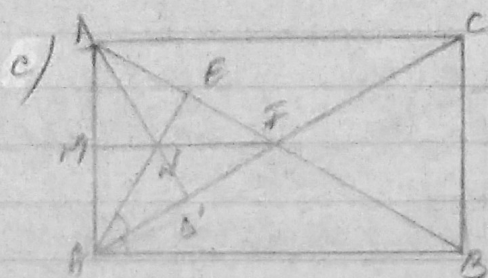
$$= \frac{(x-1)(x+5)}{x+5} \cdot \frac{1}{x-1} = 1 \cdot 1 = 1, \quad (\forall) x \in \mathbb{R}$$

$$\text{Am aplicat formula: } \boxed{a^2 - b^2 = (a-b)(a+b)}$$

$$\text{III. 1. a) } P_{ABCD} = 2(L+l) \quad \Bigg| \Rightarrow P_{ABCD} = 2(AB+AD) =$$

$$L=AB, \quad l=AD \quad \Bigg| \quad = 2(8\sqrt{3}+8) =$$

$$= 2 \cdot 8(\sqrt{3}+1) = 16(\sqrt{3}+1) \text{ cm}$$



$\triangle AFD$ echilateral.

$FN \parallel AB \quad \Bigg| \Rightarrow FN \perp AD \Rightarrow FN$ înălțime în $\triangle AFD$ ①
 $AB \perp AD$

Știm că $m(\angle DAE) = m(\angle EAF) \Rightarrow AE$ bisectoare în $\triangle AFD$ echilat
 $\Rightarrow AE \perp DF \Rightarrow AE$ înălțime ②.

Din ①, ②.

$\triangle AFD$ echilateral $\Bigg| \Rightarrow N$ ortocentrul $\triangle AFD \Rightarrow$

$\Rightarrow \triangle AN \perp DF \quad \Bigg| \Rightarrow \triangle AN \perp AF \Rightarrow \triangle AN \perp AC$
 $N \in \triangle AN \quad \Bigg| \quad F \in AC$

$$6) \triangle \text{ drept. } \triangle ABC \xrightarrow{\text{THP.}} BD^2 = AD^2 + AB^2 = (8\sqrt{3})^2 + 8^2 = 64 \cdot 3 + 64 =$$

$$= 64(3+1) = 64 \cdot 4$$

$$BD = \sqrt{64 \cdot 4} = \sqrt{8^2 \cdot 2^2} = 8 \cdot 2 = 16$$

$$AD = \frac{BD}{2} = 8 \quad \Bigg| \Rightarrow$$

$$\Rightarrow m(\widehat{ABD}) = 30^\circ \quad \Bigg| \Rightarrow m(\widehat{ADB}) = 60^\circ$$

$$m(\widehat{CAB}) = 90^\circ$$

$$m(\widehat{DAE}) = m(\widehat{EAF}) = m(\widehat{FAB}) = \frac{90^\circ}{3} = 30^\circ \Rightarrow m(\widehat{DAF}) = 60^\circ \quad \Bigg| \Rightarrow$$

$\Rightarrow \triangle DAF$ - echilateral $\Rightarrow F$ mijlocul lui $BD \quad \Bigg| \Rightarrow F \in AC$
 $ABCD$ dreptunghi

Sau:

• se mai poate arăta că A, F și C sunt coliniare (sunt pe aceeași dreaptă) dacă $m(\angle AFC) = 180^\circ$

$$2. a) A_l = 2\pi R G.$$

$$R = \frac{AB}{2} = \frac{10}{2} = 5 \text{ cm}$$

$$G = AA' = 12 \text{ cm}$$

$$A_l = 2\pi \cdot 5 \cdot 12 = 120\pi \text{ cm}^2$$

$$b) A'B < 16 \text{ cm}$$

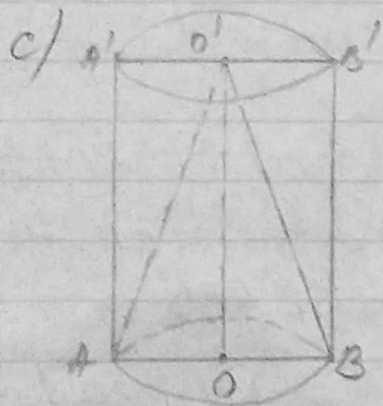
Cum în figura 3 avem un cilindru circular drept \Rightarrow
 $\Rightarrow AA' \perp AB \Rightarrow \triangle AA'B$ - dreptunghi $\xrightarrow{\text{Th. Pitagora}} A'B^2 = AA'^2 + AB^2$

$$(A'B)^2 = 12^2 + 10^2 = 144 + 100 = 244.$$

$$(A'B)^2 < 16^2$$

$$\Rightarrow 244 < 256 \Rightarrow$$

$$\Rightarrow A'B < 16$$



$$m(\angle(AO', \text{planul bazei})) =$$

$$= m(\angle(AO', AO)) = m(\angle O'AO)$$

$$OO' = AA' \quad \Bigg| \Rightarrow OO' \perp AB \Rightarrow$$

$$AA' \perp AB$$

$$\Rightarrow \triangle AOO' - \text{dreptunghi} \Rightarrow$$

$$\Rightarrow \sin(\widehat{O'AO}) = \frac{OO'}{AO'}$$

$$(AO')^2 = (OO')^2 + (AO)^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$AO = \frac{AB}{2} = \frac{10}{2} = 5 \text{ cm}$$

$$AO' = \sqrt{169} = 13 \text{ cm}$$

$$\sin(\widehat{O'AO}) = \frac{12}{13}$$