* *

5. $1 = \left(\frac{1}{2}\right)^0$

$\frac{1}{8} = \frac{1}{2^3} = \frac{1^3}{2^3} = \left(\frac{1}{2}\right)^3$

$\frac{2^{(2}}{4} = \frac{1}{2} = \left(\frac{1}{2}\right)^1$

$\frac{5}{32} = \frac{5}{2^5}$

$\frac{1}{64} = \frac{1}{2^6} = \frac{1^6}{2^6} = \left(\frac{1}{2}\right)^6.$

Numerele care se pot scrie ca puteri cu baza $\frac{1}{2}$ sunt:

$1 ; \frac{1}{8} ; \frac{2}{4} ; \frac{1}{64}$.

6. a) $\frac{343}{8} = \frac{7^3}{2^3} = \left(\frac{7}{2}\right)^3$

b) $\frac{1}{512} = \frac{1}{2^9} = \frac{1^9}{2^9} = \left(\frac{1}{2}\right)^9$

c) $\frac{125}{27} = \frac{5^3}{3^3} = \left(\frac{5}{3}\right)^3$

d) $\frac{81}{256} = \frac{3^4}{4^4} = \left(\frac{3}{4}\right)^4$

e) $1 = \left(\frac{1}{7}\right)^0$

f) $0 = 0^n$

7. a) $\left(\frac{2}{3}\right)^n = \frac{16}{81} \Leftrightarrow \left(\frac{2}{3}\right)^n = \frac{2^4}{3^4} \Leftrightarrow \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^4 \Rightarrow$

$\Rightarrow n = 4$

b) $\left(\frac{1}{7}\right)^n = \frac{1}{343} \Leftrightarrow \left(\frac{1}{7}\right)^n = \frac{1}{7^3} \Leftrightarrow \left(\frac{1}{7}\right)^n = \left(\frac{1}{7}\right)^3 \Rightarrow n = 3$

c) $\left(1\frac{1}{2}\right)^n = 3\frac{375}{1000} \Leftrightarrow \left(\frac{1\cdot 2+1}{2}\right)^n = \frac{27}{8} \Leftrightarrow \left(\frac{3}{2}\right)^n = \frac{3^3}{2^3} \Leftrightarrow$

$3\frac{375}{1000} = 3 + \frac{375}{1000}^{(125} = 3 + \frac{3}{8} = 3\frac{3}{8} = \frac{3\cdot 8+3}{8} = \frac{27}{8}$

$\Leftrightarrow \left(\frac{3}{2}\right)^n = \left(\frac{3}{2}\right)^3 \Rightarrow n = 3$

d) $\left(2\frac{1}{2}\right)^n = \frac{125}{8} \Leftrightarrow \left(\frac{2\cdot 2+1}{2}\right)^n = \frac{5^3}{2^3} \Leftrightarrow \left(\frac{5}{2}\right)^n = \left(\frac{5}{2}\right)^3 \Rightarrow$

$\Rightarrow n = 3$

e) $\left(\frac{1}{3^2}\right)^n = \frac{1}{729} \Leftrightarrow \left(\frac{1}{9}\right)^n = \frac{1}{9^3} \Leftrightarrow \left(\frac{1}{9}\right)^n = \left(\frac{1}{9}\right)^3 \Rightarrow n = 3$

f) $\left(\frac{1}{8}\right)^n = \frac{1}{4^3} \Leftrightarrow \left(\frac{1}{2^3}\right)^n = \left(\frac{1}{4}\right)^3 \Leftrightarrow \left(\frac{1}{2}\right)^{3\cdot n} = \left(\frac{1}{2^2}\right)^3 \Leftrightarrow$

$\Leftrightarrow \left(\frac{1}{2}\right)^{3\cdot n} = \left(\frac{1}{2}\right)^{2\cdot 3} \Leftrightarrow \left(\frac{1}{2}\right)^{3\cdot n} = \left(\frac{1}{2}\right)^6 \Rightarrow 3\cdot n = 6 \Rightarrow$

$\Rightarrow n = 6 : 3 \Rightarrow n = 2$

8. a) $\left[\left(\frac{5}{6}\right)^2+\left(\frac{3}{4}\right)^2\right]:\frac{181}{12^2}=\left({}^{4)}\frac{25}{36}+{}^{9)}\frac{9}{16}\right):\frac{181}{12^2}=$

N.c.=144

$=\left(\frac{100}{144}+\frac{81}{144}\right):\frac{181}{144}=\frac{181}{144}:\frac{181}{144}=\frac{\cancel{181}^{1}}{\cancel{144}_{1}}\cdot\frac{\cancel{144}^{1}}{\cancel{181}_{1}}=1.$

b) $\left[\left(\frac{1}{2}\right)^3-\left(\frac{1}{6}\right)^3\right]:\frac{128-20}{65:5}=\left({}^{27)}\frac{1}{8}-\frac{1}{216}\right):\frac{128-20}{13}=$

N.c.=216

$=\left(\frac{27}{216}-\frac{1}{216}\right):\frac{108}{13}=\frac{26}{216}:\frac{108}{13}=\frac{\cancel{26}^{13}}{\cancel{216}_{108}}\cdot\frac{13}{108}=$

$=\frac{13}{108}\cdot\frac{13}{108}=\frac{13^2}{108^2}=\left(\frac{13}{108}\right)^2$

c) $\left({}^{3)}\frac{1}{2}+{}^{2)}\frac{1}{3}+\frac{1}{6}\right)^{2011}:\left(\frac{4^{(4}}{12}\right)^2=\left(\frac{3}{6}+\frac{2}{6}+\frac{1}{6}\right)^{2011}:\left(\frac{1}{3}\right)^2=$

N.c.=6

$=\left(\frac{6}{6}\right)^{2011}:\left(\frac{1}{3}\right)^2=1^{2011}:\frac{1}{9}=1\cdot\frac{9}{1}=\frac{9}{1}=1$

d) $\left[\left({}^{3)}\frac{1}{3}-\frac{1}{9}\right)^2+\left(\frac{1}{3}\right)^3\right]:\left(\frac{1}{3}\right)^4=\left[\left(\frac{3}{9}-\frac{1}{9}\right)^2+\left(\frac{1}{3}\right)^3\right]:\left(\frac{1}{3}\right)^4=$

N.c=9

$=\left[\left(\frac{2}{9}\right)^2+\left(\frac{1}{3}\right)^3\right]:\frac{1}{81}=\left(\frac{4}{81}+{}^{3)}\frac{1}{27}\right)\cdot 81=\left(\frac{4}{81}+\frac{3}{81}\right)\cdot 81=$

N.c.=81

$=\frac{7}{\cancel{81}_{1}}\cdot\cancel{81}^{1}=7$

9. a) $\left(\frac{3}{5}\right)^{15}=p^5=q^3$

$$\left(\frac{3}{5}\right)^{15}=\left(\frac{3}{5}\right)^{3\cdot5}=\left[\left(\frac{3}{5}\right)^3\right]^5=\left(\frac{3^3}{5^3}\right)^5=\left(\frac{27}{125}\right)^5=p^5 \Rightarrow$$

$$\Rightarrow \boxed{p=\frac{27}{125}}$$

$$\left(\frac{3}{5}\right)^{15}=\left(\frac{3}{5}\right)^{3\cdot5}=\left[\left(\frac{3}{5}\right)^5\right]^3=\left(\frac{3^5}{5^5}\right)^3=\left(\frac{243}{3125}\right)^3=q^3 \Rightarrow$$

$$\Rightarrow \boxed{q=\frac{243}{3125}}$$

b) $\left(\frac{3}{7}\right)^{6}=p^2=q^3$

$$\left(\frac{3}{7}\right)^{6}=\left(\frac{3}{7}\right)^{2\cdot3}=\left[\left(\frac{3}{7}\right)^3\right]^2=\left(\frac{3^3}{7^3}\right)^2=\left(\frac{27}{343}\right)^2=p^2 \Rightarrow \boxed{p=\frac{27}{343}}$$

$$\left(\frac{3}{7}\right)^{6}=\left(\frac{3}{7}\right)^{2\cdot3}=\left[\left(\frac{3}{7}\right)^2\right]^3=\left(\frac{3^2}{7^2}\right)^3=\left(\frac{9}{49}\right)^3=q^3 \Rightarrow \boxed{q=\frac{9}{49}}$$

c) $\left(\frac{1}{2}\right)^{10}=p^5=q^2$

$$\left(\frac{1}{2}\right)^{10}=\left(\frac{1}{2}\right)^{2\cdot5}=\left[\left(\frac{1}{2}\right)^2\right]^5=\left(\frac{1^2}{2^2}\right)^5=\left(\frac{1}{4}\right)^5=p^5 \Rightarrow \boxed{p=\frac{1}{4}}$$

$$\left(\frac{1}{2}\right)^{10}=\left(\frac{1}{2}\right)^{2\cdot5}=\left[\left(\frac{1}{2}\right)^5\right]^2=\left(\frac{1^5}{2^5}\right)^2=\left(\frac{1}{32}\right)^2=q^2 \Rightarrow \boxed{q=\frac{1}{32}}$$

d) $\left(\frac{1}{3}\right)^8 = p^4 = q^2$

$$\left(\frac{1}{3}\right)^8 = \left(\frac{1}{3}\right)^{2\cdot 4} = \left[\left(\frac{1}{3}\right)^2\right]^4 = \left(\frac{1^2}{3^2}\right)^4 = \left(\frac{1}{9}\right)^4 = p^4 \Rightarrow \boxed{p = \frac{1}{9}}$$

$$\left(\frac{1}{3}\right)^8 = \left(\frac{1}{3}\right)^{2\cdot 4} = \left[\left(\frac{1}{3}\right)^4\right]^2 = \left(\frac{1^4}{3^4}\right)^2 = \left(\frac{1}{81}\right)^2 = q^2 \Rightarrow \boxed{q = \frac{1}{81}}$$

e) $\left(\frac{3}{5}\right)^{15} = p^5 = q^3$

$$\left(\frac{3}{5}\right)^{15} = \left(\frac{3}{5}\right)^{3\cdot 5} = \left[\left(\frac{3}{5}\right)^3\right]^5 = \left(\frac{3^3}{5^3}\right)^5 = \left(\frac{27}{125}\right)^5 = p^5 \Rightarrow \boxed{p = \frac{27}{125}}$$

$$\left(\frac{3}{5}\right)^{15} = \left(\frac{3}{5}\right)^{3\cdot 5} = \left[\left(\frac{3}{5}\right)^5\right]^3 = \left(\frac{3^5}{5^5}\right)^3 = \left(\frac{243}{3125}\right)^3 = q^3 \Rightarrow \boxed{q = \frac{243}{3125}}$$

f) $\left(\frac{2}{7}\right)^{24} = p^3 = q^6$

$$\left(\frac{2}{7}\right)^{24} = \left(\frac{2}{7}\right)^{3\cdot 8} = \left[\left(\frac{2}{7}\right)^8\right]^3 = p^3 \Rightarrow p = \left(\frac{2}{7}\right)^8 \Rightarrow \boxed{p = \frac{256}{5764801}}$$

$$\left(\frac{2}{7}\right)^{24} = \left(\frac{2}{7}\right)^{4\cdot 6} = \left[\left(\frac{2}{7}\right)^4\right]^6 = q^6 \Rightarrow q = \left(\frac{2}{7}\right)^4 \Rightarrow \boxed{q = \frac{16}{2401}}$$

10. a) $\left(\frac{4}{9}\right)^5$ și $\left(\frac{8}{27}\right)^4 \Rightarrow \left(\frac{2}{3}\right)^{10}$ și $\left(\frac{2}{3}\right)^{12}$

$$\left(\frac{4}{9}\right)^5 = \left(\frac{2^2}{3^2}\right)^5 = \left[\left(\frac{2}{3}\right)^2\right]^5 = \left(\frac{2}{3}\right)^{2\cdot 5} = \left(\frac{2}{3}\right)^{10}.$$

$$\left(\frac{8}{27}\right)^4 = \left(\frac{2^3}{3^3}\right)^4 = \left[\left(\frac{2}{3}\right)^3\right]^4 = \left(\frac{2}{3}\right)^{3\cdot 4} = \left(\frac{2}{3}\right)^{12}$$

b) $\left(\frac{25}{36}\right)^{11}$ și $\left(\frac{125}{216}\right)^{12} \Rightarrow \left(\frac{5}{6}\right)^{22}$ și $\left(\frac{5}{6}\right)^{36}$

$$\left(\frac{25}{36}\right)^{11} = \left(\frac{5^2}{6^2}\right)^{11} = \left[\left(\frac{5}{6}\right)^2\right]^{11} = \left(\frac{5}{6}\right)^{2\cdot 11} = \left(\frac{5}{6}\right)^{22}$$

$$\left(\frac{125}{216}\right)^{12} = \left(\frac{5^3}{6^3}\right)^{12} = \left[\left(\frac{5}{6}\right)^3\right]^{12} = \left(\frac{5}{6}\right)^{3\cdot 12} = \left(\frac{5}{6}\right)^{36}$$

c) $\left(\frac{1}{9}\right)^5$ și $\left(\frac{1}{27}\right)^8 \Rightarrow \left(\frac{1}{3}\right)^{10}$ și $\left(\frac{1}{3}\right)^{24}$

$$\left(\frac{1}{9}\right)^5 = \left(\frac{1}{3^2}\right)^5 = \left[\left(\frac{1}{3}\right)^2\right]^5 = \left(\frac{1}{3}\right)^{2\cdot 5} = \left(\frac{1}{3}\right)^{10}$$

$$\left(\frac{1}{27}\right)^8 = \left(\frac{1}{3^3}\right)^8 = \left[\left(\frac{1}{3}\right)^3\right]^8 = \left(\frac{1}{3}\right)^{3\cdot 8} = \left(\frac{1}{3}\right)^{24}$$

11. a) $\left(\frac{1}{3}\right)^{26}$ și $\left(\frac{1}{4}\right)^{39} \Rightarrow \left(\frac{1}{9}\right)^{13}$ și $\left(\frac{1}{64}\right)^{13}$

$$\left(\frac{1}{3}\right)^{26} = \left(\frac{1}{3}\right)^{2\cdot 13} = \left[\left(\frac{1}{3}\right)^2\right]^{13} = \left(\frac{1}{9}\right)^{13}$$

$$\left(\frac{1}{4}\right)^{39} = \left(\frac{1}{4}\right)^{3\cdot 13} = \left[\left(\frac{1}{4}\right)^3\right]^{13} = \left(\frac{1}{64}\right)^{13}$$

b) $\left(6\frac{1}{4}\right)^{48}$ și $\left(3\frac{1}{2}\right)^{96}$ $\Rightarrow$ $\left(\frac{5}{2}\right)^{96}$ și $\left(\frac{7}{2}\right)^{96}$

$\left(6\frac{1}{4}\right)^{48} = \left(\frac{6\cdot 4+1}{4}\right)^{48} = \left(\frac{25}{4}\right)^{48} = \left(\frac{5^2}{2^2}\right)^{48} = \left[\left(\frac{5}{2}\right)^2\right]^{48} = \left(\frac{5}{2}\right)^{2\cdot 48} = \left(\frac{5}{2}\right)^{96}$

$\left(3\frac{1}{2}\right)^{96} = \left(\frac{3\cdot 2+1}{2}\right)^{96} = \left(\frac{7}{2}\right)^{96}$.

c) $\left(\frac{1}{9}\right)^{18}$ și $\left(\frac{5}{3}\right)^{36}$ $\Rightarrow$ $\left(\frac{1}{9}\right)^{18}$ și $\left(\frac{25}{9}\right)^{18}$

$\left(\frac{5}{3}\right)^{36} = \left(\frac{5}{3}\right)^{2\cdot 18} = \left[\left(\frac{5}{3}\right)^2\right]^{18} = \left(\frac{5^2}{3^2}\right)^{18} = \left(\frac{25}{9}\right)^{18}$

12. a) $2^{17} > 4^8$

$4^8 = (2^2)^8 = 2^{2\cdot 8} = 2^{16}$

$2^{17} > 2^{16}$

b) $\left(\frac{2}{5}\right)^{21} < \left(\frac{8}{125}\right)^{8}$

$\left(\frac{8}{125}\right)^{8} = \left(\frac{2^3}{5^3}\right)^{8} = \left[\left(\frac{2}{5}\right)^3\right]^{8} = \left(\frac{2}{5}\right)^{3\cdot 8} = \left(\frac{2}{5}\right)^{24}$

$\left(\frac{2}{5}\right)^{21} < \left(\frac{2}{5}\right)^{24}$

c) $\left(\frac{3}{17}\right)^{21} < \left(\frac{1}{3}\right)^{28}$

$\left(\frac{3}{17}\right)^{21} = \left(\frac{3}{17}\right)^{3\cdot 7} = \left[\left(\frac{3}{17}\right)^3\right]^{7} = \left(\frac{3^3}{17^3}\right)^{7} = \left(\frac{27}{4913}\right)^{7}$

$\left(\frac{1}{3}\right)^{28} = \left(\frac{1}{3}\right)^{4\cdot 7} = \left[\left(\frac{1}{3}\right)^4\right]^{7} = \left(\frac{1}{3^4}\right)^{7} = \left(\frac{1}{81}\right)^{7}$

$\left(\frac{27}{4913}\right)^{7} < \left(\frac{1}{81}\right)^{7}$, deoarece $\frac{^{27)}1}{81} = \frac{27}{2187}$, deci $\frac{27}{4913} < \frac{27}{2187}$

13. a) $x \cdot x = \frac{81}{169} \Leftrightarrow x^{1+1} = \frac{9^2}{13^2} \Leftrightarrow x^2 = \left(\frac{9}{13}\right)^2 \Rightarrow x = \frac{9}{13}$

b) $x \cdot x \cdot x \cdot x = \frac{16}{10\,000} \Leftrightarrow x^4 = \frac{2^4}{10^4} \Leftrightarrow x^4 = \left(\frac{2}{10}\right)^4 \Rightarrow$

$\Rightarrow x = \frac{2}{10}^{(2} \Rightarrow x = \frac{1}{5}$

c) $x \cdot x \cdot x = \frac{1}{27} \Leftrightarrow x^3 = \frac{1}{3^3} \Leftrightarrow x^3 = \left(\frac{1}{3}\right)^3 \Rightarrow x = \frac{1}{3}$

14. a) $\frac{a}{b} = \frac{13}{12} \Leftrightarrow 12 \cdot a = 13 \cdot b$

$${}^{3)}\frac{4a+b}{4a-3b} = \frac{3(4a+b)}{3(4a-3b)} = \frac{12a+3b}{12a-9b} = \frac{13b+3b}{13b-9b} = \frac{\cancel{16b}^{\,4}}{\cancel{4b}_{\,1}} = 4 =$$

$$= 2^2 \Rightarrow \frac{4a+b}{4a-3b} = 2^2$$

b) $$\left(\frac{3}{5}\right)^k : \frac{3^{k+2}+6^{k+2}}{5^{k+2}+10^{k+2}} = \left(\frac{3}{5}\right)^k : \frac{3^{k+2}+(2\cdot 3)^{k+2}}{5^{k+2}+(5\cdot 2)^{k+2}} =$$

$k \in \mathbb{N}$

$$= \left(\frac{3}{5}\right)^k : \frac{3^{k+2}+2^{k+2}\cdot 3^{k+2}}{5^{k+2}+5^{k+2}\cdot 2^{k+2}} = \frac{3^k}{5^k} : \frac{3^{k+2}\cdot(1+2^{k+2})}{5^{k+2}\cdot(1+2^{k+2})} =$$

$$= \frac{\cancel{3^k}}{\cancel{5^k}} \cdot \frac{\cancel{5^k}\cdot 5^2 \cdot \cancel{(1+2^{k+2})}}{\cancel{3^k}\cdot 3^2 \cdot \cancel{(1+2^{k+2})}} = \frac{5^2}{3^2} = \frac{25}{9}$$