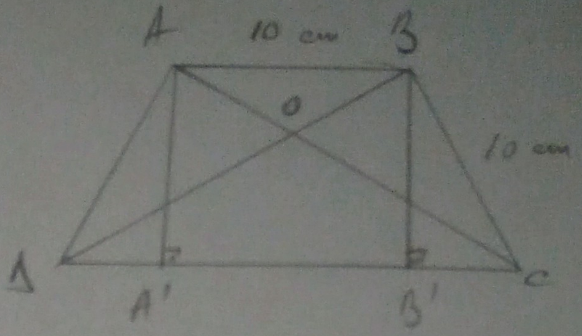
80. Ip: ABCD – trapez isoscel

$AB \parallel CD$

$AB = BC = 10$ cm

$\frac{OA}{OC} = \frac{5}{11}$, $\{O\} = AC \cap BD$



C: $A_{ABCD} = ?$

Rezolvare: $A_{ABCD} = \frac{(AB + DC) \cdot BB'}{2}$

Dacă ABCD - trapez isoscel $\Rightarrow AD = BC$, $m(\sphericalangle D) = m(\sphericalangle C)$

$\Delta AA'D \underset{u.i}{\equiv} \Delta BB'C$ $(AD \equiv BC, \sphericalangle D \equiv \sphericalangle C)$

$\Downarrow$

$DA' \equiv B'C$

$DC = 2B'C + A'B' = 2B'C + AB = 2B'C + 10$

Dacă $AB \parallel DC \Rightarrow \Delta AOB \sim \Delta COD \Rightarrow \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{DC}$

$\frac{AO}{CO} = \frac{AB}{DC} \Leftrightarrow \frac{5}{11} = \frac{10}{DC} \Rightarrow DC = \frac{110}{5} \Rightarrow DC = 22$ cm

$DC = 2B'C + 10 \Leftrightarrow 22 = 2B'C + 10 \Leftrightarrow 2B'C = 22 - 10 \Leftrightarrow$

$\Leftrightarrow 2B'C = 12 \Rightarrow B'C = 12 : 2 \Rightarrow B'C = 6$ cm

În $\Delta BB'C$ - dreptunghic

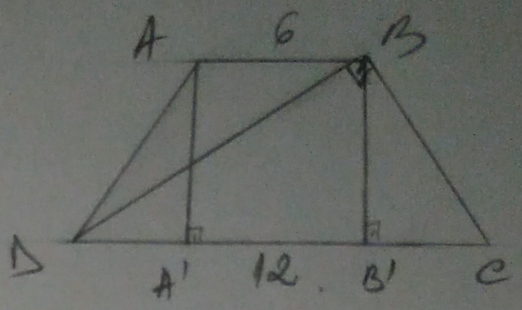
din Th. lui Pitagora $\Rightarrow BC^2 = BB'^2 + B'C^2 \Leftrightarrow 10^2 = BB'^2 + 6^2 \Leftrightarrow$

$\Leftrightarrow BB'^2 = 100 - 36 \Rightarrow BB' = \sqrt{64} \Rightarrow BB' = 8$ cm

$A_{ABCD} = \frac{(10 + 22) \cdot \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}}} = 32 \cdot 4 = 128$ cm^2

81. Ip: $ABCD$ – trapez isoscel
$AB = 6$ cm, $CD = 12$ cm
$BD \perp BC$

C: $A_{ABCD} = ?$



Rezolvare: $A_{ABCD} = \dfrac{(AB + DC) \cdot BB'}{2}$

Dacă $ABCD$ – trapez isoscel $\Rightarrow AD = BC,\ BD = AC,\ \sphericalangle D \equiv \sphericalangle C$

Ducem $BB' \perp DC$ şi $AA' \perp DC$

$\triangle AA'D \underset{u.i}{\equiv} \triangle BB'C \ (\sphericalangle D \equiv \sphericalangle C;\ AD \equiv BC) \Rightarrow$

$\Downarrow$

$DA' \equiv B'C$

$\Rightarrow DC = 2B'C + A'B' = 2B'C + AB = 2B'C + 6$

$12 = 2B'C + 6 \Leftrightarrow 2B'C = 12 - 6 \Leftrightarrow 2B'C = 6 \Rightarrow B'C = 3$ cm

$DB' = DC - B'C = 12 - 3 = 9$ cm

În $\triangle DBC$ – dreptunghic $(BD \perp BC) \overset{Th. înălţimii}{\Rightarrow} BB'^2 = DB' \cdot B'C$

$\Rightarrow BB'^2 = 9 \cdot 3 \Rightarrow BB' = \sqrt{27} \Rightarrow BB' = 3\sqrt{3}$ cm

$A_{ABCD} = \dfrac{(6+12) \cdot 3\sqrt{3}}{2} = \dfrac{\overset{9}{\cancel{18}} \cdot 3\sqrt{3}}{\cancel{2}} = 27\sqrt{3}\ \text{cm}^2$

$A_{ABCD} = 27\sqrt{3}\ \text{cm}^2$

82. Ip: ΔABC, $m(\hat{A}) = 90°$

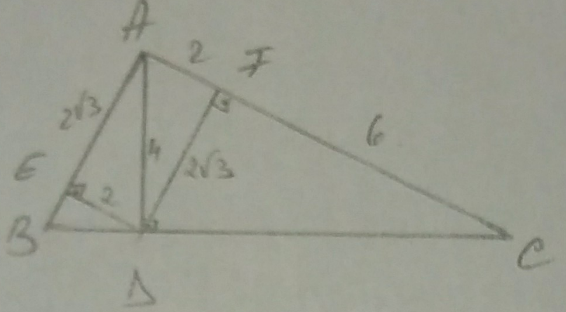
AD înățime, $pr_{AB}D = E$ și $pr_{AC}D = F$

C: a) $A_{\Delta ABC} = ?$, $DE = 2cm$, $DF = 2\sqrt{3}$ cm

b) $DE = x$, $DF = y$

$$A_{ABC} = \frac{(x^2+y^2)^2}{2xy} \; ?$$

Rezolvare:



a) Dacă $AB \perp AC$, $DF \perp AC$, $E \in AB$ $\Rightarrow DF \parallel AE \Rightarrow AEDF$ - dreptunghi $\Rightarrow$

$\Rightarrow AE = DF = 2\sqrt{3}$ cm

$DE = AF = 2$ cm

ΔAED - dreptunghic $(m(\sphericalangle E) = 90°) \xRightarrow{Th.\ Pitagora} AD^2 = DE^2 + AE^2 \Rightarrow$

$\Rightarrow AD^2 = (2\sqrt{3})^2 + 2^2 = 12 + 4 = 16 \Rightarrow AD = \sqrt{16} = 4$ cm

Din Th. înălțimii în ΔADC (dreptunghic $m(\sphericalangle D) = 90°$) $\Rightarrow$

$\Rightarrow DF^2 = AF \cdot FC \Rightarrow (2\sqrt{3})^2 = 2 \cdot FC \Rightarrow FC = \frac{12}{2} \Rightarrow FC = 6cm$

Din Th. înălțimii în ΔADB (dreptunghic $m(\sphericalangle D) = 90°$) $\Rightarrow$

$\Rightarrow DE^2 = BE \cdot EA \Rightarrow BE = 2^2 : 2\sqrt{3} = \frac{4^{2)}}{2\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ cm $\Rightarrow$

$\Rightarrow BE = \frac{2\sqrt{3}}{3}$ cm

$AC = AF + FC \Rightarrow AC = 2 + 6 = 8$ cm

$AB = BE + EA \Rightarrow AB = \frac{2\sqrt{3}}{3} + {}^{3)}2\sqrt{3} \Rightarrow AB = \frac{8\sqrt{3}}{3}$ cm

$$A_{\Delta ABC} = \frac{c_1 \cdot c_2}{2} = \frac{AB \cdot AC}{2} = \frac{\frac{8\sqrt{3}}{3} \cdot 8}{2} = \frac{\cancel{64}^{32}\sqrt{3}}{\cancel{6}_{3}} = \frac{32\sqrt{3}}{3}\ cm^2$$

b) $DE = x$, $DF = y$

Din supunctul a) $\Rightarrow AE = DF = y$

$DE = AF = x$

$AD^2 = DE^2 + AE^2 \Rightarrow AD = \sqrt{x^2+y^2}$

$FC = DF^2 : AF \Rightarrow FC = y^2 : x \Rightarrow FC = \frac{y^2}{x}$

$BE = DE^2 : EA \Rightarrow BE = x^2 : y \Rightarrow BE = \frac{x^2}{y}$

$AC = AF + FC \Rightarrow AC = {}^{x)}x + \frac{y^2}{x} \Rightarrow AC = \frac{x^2+y^2}{x}$

$AB = BE + EA \Rightarrow AB = \frac{x^2}{y} + {}^{y)}y \Rightarrow AB = \frac{x^2+y^2}{y}$

$$A_{\Delta ABC} = \frac{c_1 \cdot c_2}{2} = \frac{AB \cdot AC}{2} = \frac{\frac{x^2+y^2}{y} \cdot \frac{x^2+y^2}{x}}{2} = \frac{(x^2+y^2)^2}{2xy} \Rightarrow$$

$$\Rightarrow A_{\Delta ABC} = \frac{(x^2+y^2)^2}{2xy}$$