Tema 3.1
Divizibilitatea numerelor naturale

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1. a) $48 \vdots 16$, deoarece $48 : 16 = 3$ rest 0.

b) $125 \vdots 5$, deoarece $125 : 5 = 25$ rest 0.

c) $222 \vdots 3$, deoarece $222 : 3 = 74$ rest 0

d) $392 \vdots 7$, deoarece $392 : 7 = 56$ rest 0

e) $112 \nmid 37$, deoarece $112 : 37 = 3$ rest 1

f) $450 \vdots 9$, deoarece $450 : 9 = 50$ rest 0

g) $7 \nmid 128$, deoarece $128 : 7 = 18$ rest 2

h) $13 \mid 585$, deoarece $585 : 13 = 45$ rest 0

i) $12 \nmid 418$, deoarece $418 : 12 = 34$ rest 10

j) $60 \vdots 10$, deoarece $60 : 10 = 6$ rest 0.

k) $144 \vdots 9$, deoarece $144 : 9 = 16$ rest 0

l) $2880 \vdots 24$, deoarece $2880 : 24 = 120$ rest 0

2. a) $\Delta_{48} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

b) $\Delta_{50} = \{1, 2, 5, 10, 25, 50\}$.

c) $\Delta_{75} = \{1, 3, 5, 15, 25, 75\}$

d) $\Delta_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

e) $\Delta_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

f) $\Delta_{1001} = \{1; 7; 11; 13; 77; 91; 143; 1001\}$.

g) $\Delta_{125} = \{1, 5, 25, 125\}$

h) $\Delta_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

i) $\Delta_{91} = \{1, 7, 13, 91\}$

j) $\Delta_{19} = \{1, 19\}$

k) $\Delta_{35} = \{1, 5, 7, 35\}$

3. a) $M_7 = \{0 \cdot 7, 1 \cdot 7, 2 \cdot 7, 3 \cdot 7, 4 \cdot 7, 5 \cdot 7, 6 \cdot 7, 7 \cdot 7, 8 \cdot 7, 9 \cdot 7, 10 \cdot 7, 11 \cdot 7,$
$12 \cdot 7, 13 \cdot 7, 14 \cdot 7\} = \{0, 7, 14, 21, 28, 35, 42, 49, 56, 63,$
$70, 77, 84, 91, 98\}$

b) $M_{11} = \{0, 11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132\}$

c) $M_5 = \{25, 30, 35, 40, 45\}$

d) $M_{17} = \{3 \cdot 17, 4 \cdot 17, 5 \cdot 17, 6 \cdot 17, 7 \cdot 17, 8 \cdot 17, 9 \cdot 17, 10 \cdot 17, 11 \cdot 17\} =$
$= \{51, 68, 85, 102, 119, 136, 153, 170, 187\}$

e) $M_{13} = \{0, 1 \cdot 13, 2 \cdot 13, 3 \cdot 13, 4 \cdot 13, 5 \cdot 13, 6 \cdot 13, 7 \cdot 13\} =$
$= \{0, 13, 26, 39, 52, 65, 78, 91\}$

f) $M_{234} = \{91 \cdot 234, 2 \cdot 234, 3 \cdot 234, 4 \cdot 234\} =$
$= \{0, 234, 468, 702, 936\}$

4. a) $180 : 12 = 15 \Rightarrow 180 = 12 \cdot 15 \Rightarrow 12 \mid 180$.

$12 : 6$ deoarece $12 : 6 = 2 \Rightarrow 12 = 6 \cdot 2$

b) $984 : 8 = 123 \Rightarrow 984 = 8 \cdot 123 \Rightarrow 8 \mid 984$

$8 : 2$, deoarece $8 : 2 = 4 \Rightarrow 8 = 2 \cdot 4$

c) $3332 : 91 = 36$ rest $56 \Rightarrow 91 \nmid 3332$

$91 : 7$ deoarece $91 : 7 = 13 \Rightarrow 91 = 7 \cdot 13$

d) $700 : 35 = 20 \Rightarrow 700 = 35 \cdot 20 \Rightarrow 35 \mid 700$

$35 : 8 = 4$ rest $3 \Rightarrow 35 \nmid 8$.

e) $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 24 \cdot 5 \Rightarrow 5! : 24$.

$24 : 5 = 4$ rest $4 \Rightarrow 24 \nmid 5$

f) $14 \cdot 10 = 7 \cdot 5 \cdot 4 \Rightarrow 7 \cdot 5 \mid 14 \cdot 10$

$3 \nmid 7 \cdot 5$, deoarece $7 \cdot 5$ nu-l are ca divizor pe 3.

h) $6 \cdot 8 = 2 \cdot 3 \cdot 2 \cdot 4 = 2 \cdot 3 \cdot 4 \cdot 2 \Rightarrow 2 \cdot 3 \cdot 4 \mid 6 \cdot 8$.

$2 \cdot 3 \cdot 4 : 8 \Rightarrow 3 \cdot 8 : 8$.

i) $12 \cdot 13 : 26$. deoarece $12 \cdot 13 = 2 \cdot 6 \cdot 13 = 26 \cdot 6$.

5. a) $23 \mid 345$, deoarece $345 : 23 = 15$ rest 0

b) $6 \mid 204\,396 \Rightarrow 204\,396 : 6 = 34\,066$ rest 0

c) $1245 : 15 \Rightarrow 1245 : 15 = 83$ rest 0

d) $47 \mid 47 \Rightarrow 47 : 47 = 1$ rest 0

e) $8323 : 41 \Rightarrow 8323 : 41 = 203$ rest 0.

f) $59 \mid 12\,095 \Rightarrow 12095 : 59 = 205$ rest 0

g) $111 \mid (\overline{abc} + \overline{bca} + \overline{cab})$.

$\overline{abc} + \overline{bca} + \overline{cab} = a \cdot 100 + b \cdot 10 + c + b \cdot 100 + c \cdot 10 + a +$

$+ c \cdot 100 + a \cdot 10 + b = 100 \cdot (a+b+c) + 10 \cdot (a+b+c) + (a+b+c) =$

$= (a+b+c) \cdot (100 + 10 + 1) = 111 \cdot (a+b+c)$

$111 \mid (\overline{abc} + \overline{bca} + \overline{cab}) \Rightarrow 111 \mid 111 \cdot (a+b+c)$

h) $30^n \mid (5^n \cdot 6^n + 15^n \cdot 2^{n+1}) \Rightarrow 30^n \mid 30^n \cdot 3.$

$5^n \cdot 6^n + 15^n \cdot 2^{n+1} = (5 \cdot 6)^n + 15^n \cdot 2^n \cdot 2 = 30^n + (15 \cdot 2)^n \cdot 2 =$

$= 30^n + 30^n \cdot 2 = 30^n \cdot (1+2) = 30^n \cdot 3$

i) $27\,540 : (1^2 + 2^2 + 3^2 + 4^2) \Rightarrow 27\,540 : 30$

$1^2 + 2^2 + 3^2 + 4^2 = \underline{1} + \underline{\underline{4}} + \underline{9} + \underline{\underline{16}} = 10 + 20 = 30$

$27540 : 30 = 918$ rest 0

j) $50\,500 : (1 + 2 + 3 + \ldots + 100) \Rightarrow 50\,500 : 5050$

$1 + 2 + 3 + \ldots + 100 = 100 \cdot (100+1) : 2 = 100 \cdot 101 : 2 = 101 \cdot 50 =$

$= 5050.$

$50\,500 : 5050 = 10.$ rest 0

k) $21 \mid (1+2+3+\ldots+20) \Rightarrow 21 \mid 210$, deoarece $210 : 21 = 10$ rest 0

$1+2+3+\ldots+20 = 20\cdot(20+1):2 = 21\cdot 20:2 = 21\cdot 10 = 210.$

6. a) a, b = numere impare.

$a = 2m+1$

$b = 2n+1$

$a+b = 2m+1+2n+1 = 2m+2n+2 = 2\cdot(m+n+1)$

$(a+b) \vdots 2$ deoarece $2\cdot(m+n+1) \vdots 2$

b) a, b = două numere pare

$a = 2K$

$b = 2n$

$a+b = 2K+2n = 2\cdot(K+n)$

$(a+b) \vdots 2 \Rightarrow 2\cdot(K+n) \vdots 2.$

c) a, b = două numere naturale de paritați diferite

$a = 2K.$

$b = 2n+1$

$a+b = 2K+2n+1 = 2\cdot(K+n)+1.$

$(a+b) \not\vdots 2 \Rightarrow 2\cdot(K+n)+1 \not\vdots 2.$

7. a) $D_{15} = \{1, 3, 5, 15\}$

$15 \vdots (x+1) \Rightarrow$ $x+1 = 1 \Rightarrow x = 0.$

$x+1 = 3 \Rightarrow x = 2$

$x+1 = 5 \Rightarrow x = 4$

$x+1 = 15 \Rightarrow x = 14$

$\Rightarrow x = \{0, 2, 4, 14\}$

b) $D_{26} = \{1, 2, 13, 26\}$

$26 : (2x+1) \Rightarrow 2x+1 = 1 \Rightarrow 2x = 0 \Rightarrow x = 0.$

$2x+1 = 2 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2} \notin \mathbb{N}.$

$2x+1 = 13 \Rightarrow 2x = 12 \Rightarrow x = 6.$

$2x+1 = 26 \Rightarrow 2x = 25 \Rightarrow x = \frac{25}{2} \notin \mathbb{N}.$

$\Rightarrow x = \{0, 6\}$

c) $D_{32} = \{1, 2, 4, 8, 16, 32\}$

$32 : (2x+4) \Rightarrow 2x+4 = 1 \Rightarrow 2x = -3.$ nu convine.

$2x+4 = 2 \Rightarrow 2x = -2$ nu convine

$2x+4 = 4 \Rightarrow 2x = 0 \Rightarrow x = 0.$

$2x+4 = 8 \Rightarrow 2x = 4 \Rightarrow x = 2.$

$2x+4 = 16 \Rightarrow 2x = 12 \Rightarrow x = 6$

$2x+4 = 32 \Rightarrow 2x = 28 \Rightarrow x = 14$

$\Rightarrow x = \{0, 2, 6, 14\}.$

d) $D_{70} = \{1, 2, 5, 7, 10, 14, 35, 70\}.$

$70 : (2x+5) \Rightarrow 2x+5 = 1 \Rightarrow 2x = -4$ nu convine

$2x+5 = 2 \Rightarrow 2x = -3$ nu convine

$2x+5 = 5 \Rightarrow 2x = 0 \Rightarrow x = 0.$

$2x+5 = 7 \Rightarrow 2x = 2 \Rightarrow x = 1.$

$2x+5 = 10 \Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$ nu convine

$2x+5 = 14 \Rightarrow 2x = 9 \Rightarrow x = \frac{9}{2}$ nu convine

$2x+5 = 35 \Rightarrow 2x = 30 \Rightarrow x = 15$

$2x+5 = 70 \Rightarrow 2x = 65 \Rightarrow x = \frac{65}{2}$ nu convine

$\Rightarrow x = \{0, 1, 15\}$

e) $D_{45} = \{1, 3, 5, 9, 15, 45\}$

$45 : (3x-3) \Rightarrow 3x-3 = 1 \Rightarrow 3x = 4 \Rightarrow x = \frac{4}{3}$ nu convine

$3x-3 = 3 \Rightarrow 3x = 6 \Rightarrow x = 2$

$3x-3 = 5 \Rightarrow 3x = 8 \Rightarrow x = \frac{8}{3}$ nu convine

$3x-3 = 9 \Rightarrow 3x = 12 \Rightarrow x = 4$

$3x-3 = 15 \Rightarrow 3x = 18 \Rightarrow x = 6$

$3x-3 = 45 \Rightarrow 3x = 48 \Rightarrow x = 48 : 3 = 16$

$\Rightarrow x = \{2, 4, 6, 16\}$

8. a) $x = ?$

$(x+1) \vdots 14$, $12 \leq x \leq 100$

$M_{14} = \{14, 28, 42, 56, 70, 84, 98\}$

$(x+1) \vdots 14 \Rightarrow x+1 = 14 \Rightarrow x = 13$

$x+1 = 28 \Rightarrow x = 27$

$x+1 = 42 \Rightarrow x = 41$

$x+1 = 56 \Rightarrow x = 55$

$x+1 = 70 \Rightarrow x = 69$

$x+1 = 84 \Rightarrow x = 83$

$x+1 = 98 \Rightarrow x = 97$

$x = \{13, 27, 41, 55, 69, 83, 97\}$

b) $y = ?$

$(y-5) : 37$, $10 \leq y \leq 120$.

$M_{37} = \{0, 37, 74, 111\}$

$(y-5) : 37 \Rightarrow$ $y - 5 = 0 \Rightarrow y = 5$ nu convine ($10 \leq 5 \leq 120$)

$y - 5 = 37 \Rightarrow y = 42$

$y - 5 = 74 \Rightarrow y = 79$

$y - 5 = 111 \Rightarrow y = 116$.

$\Rightarrow y = \{42, 79, 116\}$

c) $z = ?$, $(z+4) : 24$, $10 \leq z \leq 50$.

$M_{24} = \{24, 48, \ldots\}$

$(z+4) : 24 \Rightarrow$ $z + 4 = 24 \Rightarrow z = 20$.

$z + 4 = 48 \Rightarrow z = 42$.

$\Rightarrow z = \{20, 42\}$

d) $u = ?$

$(2u+1) : 45$, $0 \leq u \leq 22$.

$M_{45} = \{45, \ldots\}$

$(2u+1) : 45 \Rightarrow 2u + 1 = 45 \Rightarrow 2u = 44 \Rightarrow u = 44 : 2 = 22$.

$\Rightarrow u = 22$

9. a) a, b = două numere consecutive

$a = 2K$

$b = 2K+1$

$a \cdot b = 2K \cdot (2K+1)$

$(a \cdot b) : 2 \Rightarrow 2K \cdot (2K+1) : 2$.

Exemplu: pentru $K=1 \Rightarrow a = 2 \cdot 2 = 4$

$b = 4+1 = 5$ $\Big| \Rightarrow a \cdot b = 4 \cdot 5$

deci, $4 \cdot 5 : 2$.

b) a, b, c = trei numere consecutive

$a = 3K$.

$b = 3K+1$.

$c = 3K+2$

$a \cdot b \cdot c = 3K \cdot (3K+1) \cdot (3K+2) \Rightarrow 3K \cdot (3K+1)(3K+2) : 3$.

Exemplu. Pentru $K=1$. avem $a = 3$

$b = 3+1 = 4$

$c = 3+2 = 5$.

$a \cdot b \cdot c = 3 \cdot 4 \cdot 5 \Rightarrow 3 \cdot 4 \cdot 5 : 3$.

c) a, b, c, d, e = cinci numere consecutive.

Fie $a = 5K$

$b = 5K+1$

$c = 5K+2$

$d = 5K+3$

$e = 5K+4$.

$\Rightarrow a \cdot b \cdot c \cdot d \cdot e = 5K \cdot (5K+1) \cdot (5K+2) \cdot (5K+3) \cdot (5K+4) \Rightarrow$

$\Rightarrow 5K \cdot (5K+1) \cdot (5K+2) \cdot (5K+3) \cdot (5K+4) : 5$

10. a, b, c – nr. naturale

a) $x = 35 \cdot a + 63 \cdot b$, $x : 7$

$x = 35 \cdot a + 63 \cdot b = 7 \cdot 5 \cdot a + 7 \cdot 9 \cdot b = 7 \cdot (5a + 9b)$

$x : 7 \Rightarrow 7 \cdot (5a + 9b) : 7$

b) $y = 48 \cdot a + 64 \cdot b$, $y : 8$

$y = 48 \cdot a + 64 \cdot b = 8 \cdot 6 \cdot a + 8 \cdot 8 \cdot b = 8 \cdot (6 \cdot a + 8 \cdot b)$

$y : 8 \Rightarrow 8 \cdot (6a + 8b) : 8$

c) $u = 5 \cdot a + 3 \cdot b + 2 \cdot c$

$v = 4 \cdot a + 6 \cdot b + 7 \cdot c$

$(u + v) : 9$

$u + v = 5a + 3b + 2c + 4a + 6b + 7c = 9a + 9b + 9c =$

$= 9 \cdot (a + b + c)$

$(u + v) : 9 \Rightarrow 9 \cdot (a + b + c) : 9$

11. $a = 1^{2013} + 2^{2013} + 3^{2013} + 4^{2013} : 2$

Presupunem că: $a = 1^1 + 2^1 + 3^1 + 4^1 = 1 + 2 + 3 + 4 = 10 : 2$

$a = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30 : 2$

$a = 1^3 + 2^3 + 3^3 + 4^3 = 1 + 8 + 27 + 64 = 100 : 2$

$a = 1^{2013} + 2^{2013} + 3^{2013} + 4^{2013} : 2.$

$\Rightarrow a : 2.$

sau.

$a : 2$ dacă a se termină în cifrele 0, 2, 4, 6 sau 8

$u(a) = u\left(1^{2013} + 2^{2013} + 3^{2013} + 4^{2013}\right) = u\left(1^{2013}\right) + u\left(2^{2013}\right)$

$+ u\left(3^{2013}\right) + u\left(4^{2013}\right) = 1 + 2 + 3 + 4 = \ldots 0 \Rightarrow$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = \ldots 6$
$2^5 = \ldots 2.$

se repetă din 4 în 4. $\Rightarrow 2013 : 4 = 503$ rest 1.

$\Rightarrow u\left(2^{2013}\right) = u\left(2^1\right) = 2.$

$3^1 = 3$
$3^2 = 9$
$3^3 = \ldots 7$
$3^4 = \ldots 1$
$3^5 = \ldots 3.$

Se repetă din 4 în 4. $\Rightarrow$ $2013 : 4 = 503$ rest 1 $\Rightarrow$

$\Rightarrow u\left(3^{2013}\right) = u\left(3^1\right) = 3$

$4^1 = 4$
$4^2 = \ldots 6$
$4^3 = \ldots 4$

se repetă din 2 în 2 $\Rightarrow 2013 : 2 = 1006$ rest 1 $\Rightarrow$

$\Rightarrow u\left(4^{2013}\right) = u\left(4^1\right) = 4.$

$\Rightarrow \left(1^{2013} + 2^{2013} + 3^{2013} + 4^{2013}\right) : 2 \Rightarrow a : 2.$

12. a) $(\overline{abbaab} - 2b) \vdots 7$

$\overline{abbaab} - 2b = a \cdot 10\,000 + b \cdot 1000 + b \cdot 100 + a \cdot 10 + b - 2b =$

$= a(10\,000 + 10) + b \cdot (1000 + 100 + 1 - 2) =$

$= a \cdot 10010 + b \cdot 1099 = 7 \cdot (1430 \cdot a + 157 \cdot b)$

$(\overline{abbaab} - 2b) \vdots \Rightarrow 7 \cdot (1430 \cdot a + 157 \cdot b) \vdots 7$

b) $(\overline{ab} + \overline{bc} + \overline{ca}) \vdots 11$

$\overline{ab} + \overline{bc} + \overline{ca} = a \cdot 10 + b + b \cdot 10 + c + c \cdot 10 + a =$

$= 10 \cdot (a + b + c) + (a + b + c) = (a + b + c) \cdot (10 + 1) =$

$= 11 \cdot (a + b + c)$

$(\overline{ab} + \overline{bc} + \overline{ca}) \vdots 11 \Rightarrow 11 \cdot (a + b + c) \vdots 11$

c) $(\overline{1a1} - \overline{1a} + 8) \vdots 9$

$\overline{1a1} - \overline{1a} + 8 = 100 + a \cdot 10 + 1 - (10 + a) + 8 = 101 + a \cdot 10 -$

$- 10 - a + 8 = 99 + 9 \cdot a = 9 \cdot (11 + a)$

$(\overline{101} - \overline{1a} + 8) \vdots 9 \Rightarrow 9 \cdot (11 + a) \vdots 9$

d) $(\overline{ab} + \overline{ba} + \overline{ac} + \overline{ca} + \overline{bc} + \overline{cb}) \vdots 11$

$\overline{ab} + \overline{ba} + \overline{ac} + \overline{ca} + \overline{bc} + \overline{cb} = a \cdot 10 + \underline{b} + b \cdot 10 + \underline{a} +$

$+ a \cdot 10 + c + c \cdot 10 + \underline{a} + b \cdot 10 + c + c \cdot 10 + \underline{b} = 10 \cdot (a + b + a + c +$

$+ b + c) + 2b + 2a + 2c = 10 \cdot (2a + 2b + 2c) + 2b + 2a + 2b =$

$= (2a + 2b + 2c) \cdot (10 + 1) = 11 \cdot (2a + 2b + 2c)$

$(\overline{ab} + \overline{ba} + \overline{ac} + \overline{ca} + \overline{bc} + \overline{cb}) \vdots 11 \Rightarrow 11 \cdot (2a + 2b + 2c) \vdots 11$