

$$16. A(x, m) = \begin{pmatrix} 2 & x & 1 \\ x & -1 & 2 \\ 3 & x & m \end{pmatrix}, x, m \in \mathbb{R}$$

$$a) \det(A(-1, 3)) = \begin{vmatrix} 2 & -1 & 1 \\ -1 & -1 & 2 \\ 3 & -1 & 3 \end{vmatrix} = -6 + 1 - 6 + 3 - 3 + 4 = -7$$

$$b) \det(A(x, m)) = \begin{vmatrix} 2 & x & 1 \\ x & -1 & 2 \\ 3 & x & m \end{vmatrix} = -2m + x^2 + 6x + 3 - mx^2 - 4x = (1-m)x^2 + 2x + 3 - 2m$$

$$c) m = ?, \det(A) \neq 0, (\forall) x \in \mathbb{R}$$

$$\det(A) = 0 \Leftrightarrow (1-m)x^2 + 2x + 3 - 2m = 0.$$

$$\Delta = 4 - 4(1-m)(3-2m) = 4 - 4(3 - 2m - 3m + 2m^2) = 4 - 12 + 20m - 8m^2 = -8m^2 + 20m - 8.$$

$$\text{Pentru } \Delta = 0 \Rightarrow -8m^2 + 20m - 8 = 0 \quad /: (-4) \Leftrightarrow$$

$$\Rightarrow 2m^2 - 5m + 2 = 0 \Leftrightarrow 2m^2 - 4m - m + 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2m(m-2) - (m-2) = 0 \Leftrightarrow (m-2)(2m-1) = 0 \Leftrightarrow$$

$$\Rightarrow \begin{cases} m-2=0 \\ 2m-1=0 \end{cases} \Rightarrow \begin{cases} m_1=2 \\ m_2=\frac{1}{2} \end{cases}$$

$$\text{Pentru } m = \left\{ \frac{1}{2}, 2 \right\} \Rightarrow \det(A) \neq 0.$$

$$17. A = \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} \in M_2(\mathbb{R}), A^n = \underbrace{A \cdot A \cdot \dots \cdot A}_n, n \in \mathbb{N}^+$$

$$a) A + A^2 = 2A$$

$$A^2 = A \cdot A = \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 16-12 & -24+18 \\ 8-6 & -12+9 \end{pmatrix} = \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix}$$

$$A + A^2 = \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 8 & -12 \\ 4 & -6 \end{pmatrix} = 2 \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} = 2A$$

$$b) X \in M_2(\mathbb{R}), X = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}, \det(X+A) = 2$$

$$X+A = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} + \begin{pmatrix} 4 & -6 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} x+4 & -6 \\ 2 & x-3 \end{pmatrix}$$

$$\det(X+A) = \begin{vmatrix} x+4 & -6 \\ 2 & x-3 \end{vmatrix} = (x+4)(x-3) + 12$$

$$\det(X+A) = 2 \Leftrightarrow (x+4)(x-3) + 12 = 2 \Leftrightarrow x^2 - 3x + 4x - 12 + 12 = 2 \Leftrightarrow x^2 + x - 2 = 0$$

$$\Leftrightarrow x^2 + x - 2 = 0$$

$$\Delta = 1 - 4 \cdot (-2) = 9$$

$$x_{1,2} = \frac{-1 \pm 3}{2} \begin{cases} \rightarrow x_1 = \frac{2}{2} = 1 \\ \rightarrow x_2 = \frac{-4}{2} = -2 \end{cases}$$

Deci: pentru $x = 1 \Rightarrow X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

pentru $x = -2 \Rightarrow X = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = -2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$c) A^m = A, \forall m \in \mathbb{N}^+$$

$$A + 2A^2 + \dots + nA^m = \frac{n(n+1)}{2} \cdot A, \forall m \in \mathbb{N}^+$$

$$A + 2A^2 + \dots + nA^m = A + 2A + \dots + nA = A(1 + 2 + \dots + n) =$$

$$= A \cdot \frac{n(n+1)}{2}$$