6. $\Delta(x) = \begin{vmatrix} 1 & x & x^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = 4 + x^2 + 16x - 4x^2 - 16 - x = -3x^2 + 15x - 12$

a) $\Delta(4) = \begin{vmatrix} 1 & 4 & 4^2 \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix} = \cancel{4} + \cancel{16} + \cancel{4 \cdot 16} - \cancel{4 \cdot 16} - \cancel{4} - \cancel{16} = 0$

b) $\Delta(x) = 0 \Leftrightarrow -3x^2 + 15x - 12 = 0 \overset{\cdot(-1)}{\Leftrightarrow} 3x^2 - 15x + 12 = 0 \Leftrightarrow$
$\overset{(:3)}{\Leftrightarrow} x^2 - 5x + 4 = 0 \Leftrightarrow x^2 - x - 4x + 4 = 0 \Leftrightarrow$
$\Leftrightarrow x(x-1) - 4(x-1) = 0 \Leftrightarrow (x-1)(x-4) = 0 \Rightarrow$
$\Rightarrow x - 1 = 0$ și $x - 4 = 0$
$\Rightarrow x_1 = 1$ și $x_2 = 4$

c) $\Delta(2^x) = 0 \Leftrightarrow -3 \cdot (2^x)^2 + 15 \cdot 2^x - 12 = 0$ $\Big/ \Rightarrow$
notăm cu $X = 2^x, X > 0$

$\Rightarrow -3X^2 + 15X - 12 = 0 \overset{b)}{\Rightarrow} X_1 = 1$ și $X_2 = 4$.

Pentru $X_1 = 1 \Rightarrow 2^x = 1 \Rightarrow 2^x = 2^0 \Rightarrow x = 0$.
Pentru $X_2 = 4 \Rightarrow 2^x = 4 \Rightarrow 2^x = 2^2 \Rightarrow x = 2$.

7. $d = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix}$, unde x_1, x_2, x_3 soluțiile ec. $x^3 - 3x + 2 = 0$

a) $x_1 + x_2 + x_3 = ?$
Din relațiile lui Viete $\Rightarrow \begin{cases} x_1 + x_2 + x_3 = -\frac{b}{a} = \frac{0}{1} = 0 \\ x_1x_2 + x_2x_3 + x_1x_3 = \frac{c}{a} = \frac{-3}{1} = -3 \\ x_1x_2x_3 = -\frac{d}{a} = -\frac{2}{1} = -2 \end{cases}$

Deci $x_1 + x_2 + x_3 = 0$

b) $x_1^3 + x_2^3 + x_3^3 = -6$?

$(x_1 + x_2 + x_3)^3 = (x_1^3 + x_2^3 + x_3^3) + 3(x_1x_2 + x_2x_3 + x_1x_3)(x_1 + x_2 + x_3) +$
$+ 3x_1x_2x_3$

Cum x_1, x_2, x_3 sunt soluțiile ecuației $x^3 - 3x + 2 = 0 \Rightarrow$

$$\Rightarrow \begin{cases} x_1^3 - 3x_1 + 2 = 0 \\ x_2^3 - 3x_2 + 2 = 0 \\ x_3^3 - 3x_3 + 2 = 0 \end{cases}$$

$+$

$x_1^3 + x_2^3 + x_3^3 - 3(x_1 + x_2 + x_3) + 6 = 0 \Rightarrow$

$\Rightarrow x_1^3 + x_2^3 + x_3^3 = 3\overbrace{(x_1 + x_2 + x_3)}^{=0} - 6 \Rightarrow$

$\Rightarrow x_1^3 + x_2^3 + x_3^3 = -6$

c) $d = ?$

$$d = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix} = x_1x_2x_3 + x_1x_2x_3 + x_1x_2x_3 - x_3^3 - x_2^3 - x_1^3 =$$

$= 3x_1x_2x_3 - (x_1^3 + x_2^3 + x_3^3) = 3\cdot(-2) - (-6) =$

$= -\cancel{6} + \cancel{6} = 0.$

9. $X = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$; $Y = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$, $A = X \cdot {}^tY$, $B(a) = aA + I_3$, $a \in \mathbb{R}$

a) $A = X \cdot {}^tY = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} \cdot (1 \ 3 \ 4) = \begin{pmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ -4 & -12 & -16 \end{pmatrix}$

b) $\det A = \begin{vmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ -4 & -12 & -16 \end{vmatrix} = -\cancel{9\cdot 16} - \cancel{12\cdot 12} - 12\cdot 12 + \cancel{9\cdot 16} + 9\cdot 16 +$
$+ \cancel{12\cdot 12} = -\cancel{144} + \cancel{144} = 0$

c) $B(a) = aA + I_3 = a\begin{pmatrix}1 & 3 & 4\\ 3 & 9 & 12\\ -4 & -12 & -16\end{pmatrix} + \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} =$

$= \begin{pmatrix}a & 3a & 4a\\ 3a & 9a & 12a\\ -4a & -12a & -16a\end{pmatrix} + \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} = \begin{pmatrix}a+1 & 3a & 4a\\ 3a & 9a+1 & 12a\\ -4a & -12a & -16a+1\end{pmatrix}$

$B(-1) = \begin{vmatrix}-1+1 & -3 & -4\\ -3 & -9+1 & -12\\ 4 & 12 & 16+1\end{vmatrix} = \begin{vmatrix}0 & -3 & -4\\ -3 & -8 & -12\\ 4 & 12 & 17\end{vmatrix} =$

$= 0 + 12\cdot 12 + 12\cdot 12 - 16\cdot 8 - 9\cdot 17 + 0 = 144 + 144 - 128 - 153 =$
$= 288 - 281 = 7 \neq 0.$

11. $\Delta(a,b,x) = \begin{vmatrix}1 & x & ab\\ 1 & a & bx\\ 1 & a & ax\end{vmatrix}, \quad a,b,x \in \mathbb{R}$

a) $\Delta(1,1,0) = \begin{vmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 1 & 1 & 0\end{vmatrix} = 0 + \cancel{1} + 0 - \cancel{1} - 0 - 0 = 0$

b) $\Delta(a,a,x) = \begin{vmatrix}1 & x & a^2\\ 1 & a & ax\\ 1 & a & ax\end{vmatrix} = \cancel{a^2x} + \cancel{a^3} + \cancel{ax^2} - \cancel{a^3} - \cancel{a^2x} - \cancel{ax^2} = 0$

c) $\Delta(a,b,x) = 0 \Leftrightarrow a^2x + \cancel{a^2b} + bx^2 - \cancel{a^2b} - ax^2 - abx = 0 \Leftrightarrow$

$\Leftrightarrow (b-a)x^2 + (a^2 - ab)x = 0 \Leftrightarrow$

$\Leftrightarrow (b-a)x^2 - a(b-a)x = 0 \Leftrightarrow$

$\Leftrightarrow (b-a)\cdot x\cdot(x-a) = 0 \Rightarrow$

$\Rightarrow \begin{cases}(b-a)x = 0\\ x - a = 0\end{cases} \Rightarrow \begin{cases}x_1 = 0\\ x_2 = a\end{cases}$