Metoda schimbării de variabilă

1. $I = \int (5x-3)^5\,dx = \frac{1}{5}\int \underbrace{5(5x-3)^5\,dx}_{dt} = \frac{(5x-3)^6}{30} + C.$

$5x+3 = t$

$(5x+3)' = t' \Rightarrow 5\,dx = dt.$

$I' = \frac{1}{5}\int t^5\,dt = \frac{1}{5}\cdot\frac{t^6}{6} + C = \frac{t^6}{30} + C.$

2. $I = \int (7x+1)^{10}\,dx = \frac{1}{7}\int 7\cdot(7x+1)^{10}\,dx = \frac{(7x+1)^{11}}{77} + C.$

$7x+1 = t$

$7\,dx = dt.$

$I' = \frac{1}{7}\int t^{10}\cdot dt = \frac{1}{7}\cdot\frac{t^{11}}{11} + C = \frac{t^{11}}{77} + C.$

3. $I = \int (2x+7)^{101}\,dx = \frac{1}{2}\int 2(2x+7)^{101}\cdot dx = \frac{(2x+7)^{102}}{204} + C.$

$2x+7 = t$

$2\,dx = dt.$

$I' = \frac{1}{2}\int t^{101}\,dt = \frac{1}{2}\cdot\frac{t^{102}}{102} + C = \frac{t^{102}}{204} + C$

4. $J=\int(5x-1)^3\,dx=\frac{1}{5}\int 5(5x-1)^3\,dx=\frac{(5x-1)^4}{20}+C$

$5x-1=t$

$5\,dx=dt$

$J'=\frac{1}{5}\int t^3\cdot dt=\frac{1}{5}\cdot\frac{t^4}{4}+C=\frac{t^4}{20}+C$

5. $J=\int(8x+3)^9\,dx=\frac{1}{8}\int 8(8x+3)^9\,dx=\frac{(8x+3)^{10}}{80}+C.$

$8x+3=t$

$8\,dx=dt.$

$J'=\frac{1}{8}\int t^9\,dt=\frac{1}{8}\cdot\frac{t^{10}}{10}+C=\frac{t^{10}}{80}+C.$

6. $J=\int 2x(x^2+7)^{103}\,dx=\frac{(x^2+7)^{104}}{104}+C$

$x^2+7=t$

$(x^2+7)'=t' \Rightarrow 2x\,dx=dt.$

$J'=\int t^{103}\,dt=\frac{t^{104}}{104}+C$

7. $J=\int 2x(2x^2-5)^{13}\,dx=\frac{1}{2}\int 4x(2x^2-5)^{13}\,dx=\frac{(2x^2-5)^{14}}{28}+C.$

$2x^2-5=t$

$(2x^2-5)'=t' \Rightarrow 4x\,dx=dt.$

$J'=\frac{1}{2}\int t^{13}\,dt=\frac{1}{2}\cdot\frac{t^{14}}{14}+C=\frac{t^{14}}{28}+C$

8. $J = \int (2x+1)(x^2+x-3)^9 dx = \frac{(x^2+x-3)^{10}}{10} + C.$

$x^2+x-3 = t$

$(x^2+x-3)'_x = (t)'_t \Rightarrow (2x+1)dx = dt$

$J' = \int t^9 dt = \frac{t^{10}}{10} + C$

9. $J = \int \sin(2x+3)\,dx = \frac{1}{2}\int 2\sin(2x+3)\,dx = -\frac{\cos(2x+3)}{2} + C$

$2x+3 = t$

$2\,dx = dt.$

$J' = \frac{1}{2}\int \sin t\,dt = \frac{1}{2}(-\cos t) + C = -\frac{\cos t}{2} + C.$

10. $J = \int \cos(7x-1)\,dx = \frac{1}{7}\int 7\cdot\cos(7x-1)\,dx = \frac{1}{7}\cdot\sin(7x-1) + C$

$7x-1 = t$

$7\,dx = dt$

$J' = \frac{1}{7}\int \cos t\,dt = \frac{1}{7}\sin t + C$

11. $I = \int \sin^3 x \, dx = \int \sin^2 x \sin x \, dx = \int (1-\cos^2 x) \sin x \, dx$

$\cos x = t$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$. $\quad -\sin x \, dx = dt$.

$$I' = -\int (1-t^2)\, dt = -\left(\int dt - \int t^2 dt\right) = -t + \frac{t^3}{3} + C$$

$$I = \frac{\cos^3 x}{3} - \cos x + C.$$

12 $I = \int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx = \int (1-\sin^2 x)\cos x \, dx$

$\sin x = t$

$\cos^2 x = 1 - \sin^2 x$.

$\cos x \, dx = dt$

$$I' = \int (1-t^2)\, dt = \int dt - \int t^2 dt = t - \frac{t^3}{3} + C$$

$$I = \sin x - \frac{\sin^3 x}{3} + C.$$

13. $I = \int \frac{\ln x}{x} dx = \frac{\ln^2 x}{2} + C$.

$\ln x = t$

$\frac{1}{x} dx = dt$

$$I' = \int t \, dt = \frac{t^2}{2} + C.$$

14. $J = \int \frac{\ln^{101} x}{x} dx = \frac{\ln^{102} x}{102} + C.$

$\ln x = t$

$\frac{1}{x} dx = dt.$

$J' = \int t^{101} dt = \frac{t^{102}}{102} + C.$

15. $J = \int \frac{\ln x + 1}{x} dx = \frac{(\ln x + 1)^2}{2} + C.$

$\ln x + 1 = t$

$\frac{1}{x} dx = dt$

$J' = \int t \, dt = \frac{t^2}{2} + C$

16. $J = \int \frac{3\ln^2 x + 5\ln x - 2}{x} dx = \int \frac{3\ln^2 x}{x} dx + \int \frac{5\ln x}{x} dx -$

$- \int \frac{2\,dx}{x} = 3\int \frac{\ln^2 x}{x} dx + 5\int \frac{\ln x}{x} dx - 2\int \frac{dx}{x} =$

$\ln x = t$

$\frac{1}{x} dx = dt$

$= \cancel{3} \cdot \frac{\ln^3 x}{\cancel{3}} + 5\frac{\ln^2 x}{2} - 2\ln x + C = \ln^3 x + \frac{5}{2}\ln^2 x - 2\ln x + C$

$J' = 3\int t^2 dt + 5\int t \, dt - 2\int dt = 3\frac{t^3}{3} + 5\frac{t^2}{2} - 2t + C.$

17. $I = \int \sin(x+3)\cos(x+3)\,dx = \frac{\sin^2(x+3)}{2} + C.$

$u = x+3.$

$du = dx.$

$I' = \int \sin u \cos u \, du = \frac{\sin^2 u}{2} + C.$

$\sin u = t$

$\cos u \, du = dt$

$I'' = \int t \, dt = \frac{t^2}{2} + C.$

18. $I = \int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \sin x \cos^2 x \, dx =$

$\sin^2 x = 1 - \cos^2 x$

$= \int (1-\cos^2 x) \underbrace{\sin x \cos^2 x \, dx}_{dt.} = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C.$

$\cos x = t$

$-\sin x \, dx = dt$

$I' = -\int (1-t^2)\cdot t^2 \, dt = -\left(\int t^2 dt - \int t^4 dt\right) =$

$= -\left(\frac{t^3}{3} - \frac{t^5}{5}\right) + C = -\frac{t^3}{3} + \frac{t^5}{5} + C.$

19. $J = \int \cos^3 x \sin^2 x \, dx = \int \cos^2 x \cdot \cos x \sin^2 x \, dx =$

$\cos^2 x = 1 - \sin^2 x.$

$= \int (1 - \sin^2 x) \cdot \underbrace{\cos x \cdot \sin^2 x \, dx}_{dt.} = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C.$

$\sin x = t.$

$\cos x \, dx = dt$

$J' = \int (1 - t^2) \cdot t^2 . \, dt = \int (t^2 - t^4) \, dt = \int t^2 dt - \int t^4 dt =$

$= \frac{t^3}{3} - \frac{t^5}{5} + C.$

20. $J = \int \frac{e^{\operatorname{tg} x}}{\cos^2 x} \, dx = e^{\operatorname{tg} x} + C.$

$\operatorname{tg} x = t$

$\frac{1}{\cos^2 x} \, dx = dt$

$J' = \int e^t \, dt = e^t + C.$