Lecția 11

1. a) $25^{27} < 25^{28}$

   b) $26^{123} < 26^{1234}$

   c) $2011^{3} < 2011^{5}$

   d) $393^{23} < 393^{100}$

   e) $125^{125} < 125^{126}$

   f) $111^{44} > 111^{33}$

2. a) $15^{27} < 17^{27}$

   b) $26^{123} > 24^{123}$

   c) $2011^{14} > 2010^{14}$

   d) $989^{123} > 987^{123}$

   e) $25^{125} < 225^{125}$

   f) $1010^{201} < 1011^{201}$

3. a) $5^{87} > 25^{36}$

   $25^{36} = (5^2)^{36} = 5^{2 \cdot 36} = 5^{72}$

   b) $4^{333} > 8^{122}$

   $4^{333} = (2^2)^{333} = 2^{2 \cdot 333} = 2^{666}$

   $8^{123} = (2^3)^{123} = 2^{3 \cdot 123} = 2^{369}$

   c) $2^{65} < 16^{20}$

   $16^{20} = (2^4)^{20} = 2^{4 \cdot 20} = 2^{80}$

d) $125^{34} < 25^{75}$

$125^{34} = (5^3)^{34} = 5^{3 \cdot 34} = 5^{102}$

$25^{75} = (5^2)^{75} = 5^{2 \cdot 75} = 5^{150}$

e) $36^{224} > 6^{363}$

$36^{224} = (6^2)^{224} = 6^{2 \cdot 224} = 6^{448}$

f) $27^{303} < 9^{502}$

$27^{303} = (3^3)^{303} = 3^{3 \cdot 303} = 3^{909}$

$9^{502} = (3^2)^{502} = 3^{2 \cdot 502} = 3^{1004}$

4. a) $3^{22} > 2^{33}$

$3^{22} = 3^{2 \cdot 11} = (3^2)^{11} = 9^{11}$

$2^{33} = 2^{3 \cdot 11} = (2^3)^{11} = 8^{11}$

b) $4^{33} < 3^{44}$

$4^{33} = 4^{3 \cdot 11} = (4^3)^{11} = 64^{11}$

$3^{44} = 3^{4 \cdot 11} = (3^4)^{11} = 81^{11}$

c) $11^{22} > 22^{11}$

$11^{22} = 11^{2 \cdot 11} = (11^2)^{11} = 121^{11}$

d) $2^{39} < 3^{26}$

$2^{39} = 2^{3 \cdot 13} = (2^3)^{13} = 8^{13}$

$3^{26} = 3^{2 \cdot 13} = (3^2)^{13} = 9^{13}$

c) $5^{45} > 6^{30}$

$5^{45} = 5^{3 \cdot 15} = (5^3)^{15} = 125^{15}$

$6^{30} = 6^{2 \cdot 15} = (6^2)^{15} = 36^{15}$

d) $15^{90} > 6^{135}$

$15^{90} = 15^{2 \cdot 45} = (15^2)^{45} = 225^{45}$

$6^{135} = 6^{3 \cdot 45} = (6^3)^{45} = 216^{45}$

5.

A	B
$2^a < 2^b$	$a = 5, b = 3$
$a^{21} > b^{21}$	$a = 8, b = 8$
$4^a > 2^b$	$a = 3, b = 7$
	$a = 4, b = 6$

$2^a < 2^b \Rightarrow a < b$

$a^{21} > b^{21} \Rightarrow a > b$

$4^a > 2^b \Rightarrow (2^2)^a > 2^b \Rightarrow 2^{2a} > 2^b \Rightarrow 2a > b$

6. a) $\overline{ab} = ?$; $12^{\overline{ab}} > 12^{97}$

$12^{\overline{ab}} > 12^{97} \Rightarrow \overline{ab} > 97 \Rightarrow \overline{ab} = \{98, 99\}$

b) $\overline{ab}$; $\overline{ab}^{12} < 13^{12}$

$\overline{ab}^{12} < 13^{12} \Rightarrow \overline{ab} < 13 \Rightarrow \overline{ab} = \{10, 11, 12\}$

c) $\overline{abc}$; $2^{\overline{abc}} < 64^{22}$

$2^{\overline{abc}} < 64^{22} \Rightarrow 2^{\overline{abc}} < 2^{132} \Rightarrow \overline{abc} < 132$

$64^{22} = (2^6)^{22} = 2^{6 \cdot 22} = 2^{132}$

$\overline{abc} = \{100; 101; 102; \ldots; 130; 131\}$

7. a) 25^{18} ; 125^{15} ; 5^{40}.

$25^{18} = (5^2)^{18} = 5^{2 \cdot 18} = 5^{36}$

$125^{15} = (5^3)^{15} = 5^{3 \cdot 15} = 5^{45}$

$5^{40} = 5^{40}$

$\nearrow$ 25^{18} ; 5^{40} ; 125^{15}

b) 9^{51} ; 27^{48} ; 3^{95}

$9^{51} = (3^2)^{51} = 3^{2 \cdot 51} = 3^{102}$

$27^{48} = (3^3)^{48} = 3^{3 \cdot 48} = 3^{144}$

$3^{95} = 3^{95}$

$\nearrow$ 3^{95} ; 9^{51} ; 27^{48}

c) 8^{12} ; 32^7 ; 27^8

$8^{12} = (2^3)^{12} = 2^{3 \cdot 12} = 2^{36}$

$32^7 = (2^5)^7 = 2^{5 \cdot 7} = 2^{35}$

$27^8 = (3^3)^8 = 3^{3 \cdot 8} = 3^{24}$

$35 < 36$.

$\log_2 3 > 2$. $\Rightarrow 2^{36} < 3^{24} \Rightarrow 2^{35} < 2^{36} < 3^{24}$

$\nearrow$ 32^7 ; 8^{12} ; 27^8

Minitest

1. a) $2^{51} > 2^{18}$, $51 > 18$

   b) $7^{32} < 9^{32}$, $7 < 9$

2. $\overline{ab}$, $25^{\overline{ab}} < 5^{26}$

   $25^{\overline{ab}} < 5^{26} \Rightarrow 25^{\overline{ab}} < 5^{2 \cdot 13} \Rightarrow 25^{\overline{ab}} < (5^2)^{13} \Rightarrow$

   $\Rightarrow 25^{\overline{ab}} < 25^{13} \Rightarrow \overline{ab} < 13 \Rightarrow \overline{ab} = \{10; 11; 12\}$

3. 4^{32} ; 8^{24} ; 16^{25}

   $4^{32} = (2^2)^{32} = 2^{2 \cdot 32} = 2^{64}$

   $8^{24} = (2^3)^{24} = 2^{3 \cdot 24} = 2^{72}$

   $16^{25} = (2^4)^{25} = 2^{4 \cdot 25} = 2^{100}$

   $\uparrow \; 4^{32}$; 8^{24} ; 16^{25}

4. $5^{72} > 3^{96}$

   $5^{72} = 5^{3 \cdot 24} = (5^3)^{24} = 125^{24}$

   $3^{96} = 3^{4 \cdot 24} = (3^4)^{24} = 81^{24}$