Lectia 12

1. a) $812 = 8\cdot 100 + 1\cdot 10 + 2 = 8\cdot 10^2 + 1\cdot 10 + 7$

b) $1121 = 1\cdot 1000 + 1\cdot 100 + 2\cdot 10 + 1 = 1\cdot 10^3 + 1\cdot 10^2 + 2\cdot 10 + 1$

c) $67\,008 = 6\cdot 10\,000 + 7\cdot 1000 + 8 = 6\cdot 10^4 + 7\cdot 10^3 + 8$

d) $12\,014 = 1\cdot 10\,000 + 2\cdot 1000 + 1\cdot 10 + 4 = 1\cdot 10^4 + 2\cdot 10^3 + 1\cdot 10 + 4$

e) $\overline{a7b} = a\cdot 100 + 7\cdot 10 + b = a\cdot 10^2 + 7\cdot 10 + b$

f) $\overline{a19b} = a\cdot 1000 + 1\cdot 100 + 9\cdot 10 + b = a\cdot 10^3 + 1\cdot 10^2 + 9\cdot 10 + b$

2. a) $4\cdot 10^3 + 7\cdot 10^2 + 8 = 4\cdot 1000 + 7\cdot 100 + 8 = 4\,708$

b) $6\cdot 10^4 + 4\cdot 10^3 + 7\cdot 10^2 + 9\cdot 10 = 6\cdot 10\,000 + 4\cdot 1000 + 7\cdot 100 + 9\cdot 10 =$
$= 64\,790$

c) $8\cdot 10^3 + 9\cdot 10 + 5 = 8\cdot 1000 + 9\cdot 10 + 5 = 8\,095$

d) $7\cdot 10^5 + 4\cdot 10^3 + 8\cdot 10^2 + 3 = 7\cdot 100000 + 4\cdot 1000 + 8\cdot 100 + 3 = 704\,803$

e) $4\cdot 10^3 + 9\cdot 10^2 + 6 = 4\cdot 1000 + 9\cdot 100 + 6 = 4\,906$

f) $5\cdot 10^2 + 9\cdot 10 + 8 = 5\cdot 100 + 9\cdot 10 + 8 = 598$

3. a) $101_{(2)} = 1\cdot 2^2 + 1 = 4 + 1 = 5_{(10)}$

b) $1101_{(2)} = 1\cdot 2^3 + 1\cdot 2^2 + 1 = 8 + 4 + 1 = 13_{(10)}$

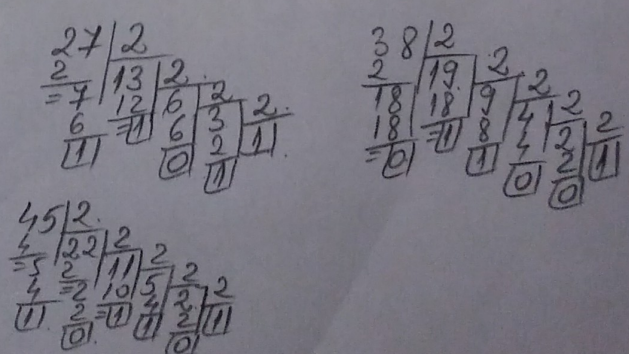
c) $10110_{(2)} = 1\cdot 2^4 + 1\cdot 2^2 + 1\cdot 2 = 16 + 4 + 2 = 22_{(10)}$

d) $11001_{(2)} = 1\cdot 2^4 + 1\cdot 2^3 + 1 = 16 + 8 + 1 = 25_{(10)}$

4. a) $27 = 11011_{(2)}$

b) $38 = 100110_{(2)}$

c) $45 = 101101_{(2)}$



d) $57 = 111001_{(2)}$

e) $61 = 111101_{(2)}$

f) $72 = 1001000_{(2)}$

g) $85 = 1010101_{(2)}$

h) $97 = 1100001_{(2)}$

$$\begin{array}{l|l} 57 & 2 \\ 56 & 28 \\ =1 & 2 \\ & =8 \\ & 8 \\ & =0 \end{array} \quad \begin{array}{l|l} 28 & 2 \\ 14 & 14 \\ =0 & 7 \\ & 6 \\ & =1 \end{array} \quad \begin{array}{l|l} 14 & 2 \\ 7 & 6 \\ =1 & 3 \\ & 2 \\ & =1 \end{array} \quad \begin{array}{l|l} 7 & 2 \\ 6 & 3 \\ =1 & 2 \\ & =1 \end{array}$$

$$\begin{array}{l|l} 61 & 2 \\ 60 & 30 \\ =1 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 30 & 2 \\ 30 & 15 \\ =0 & 14 \\ & =1 \end{array} \quad \begin{array}{l|l} 15 & 2 \\ 14 & 7 \\ =1 & 6 \\ & =1 \end{array} \quad \begin{array}{l|l} 7 & 2 \\ 6 & 3 \\ =1 & 2 \\ & =1 \end{array}$$

$$\begin{array}{l|l} 72 & 2 \\ 6 & 36 \\ 12 & 2 \\ 12 & 36 \\ =0 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 36 & 2 \\ 36 & 18 \\ =0 & 18 \\ & =0 \end{array} \quad \begin{array}{l|l} 18 & 2 \\ 18 & 9 \\ =0 & 8 \\ & =1 \end{array} \quad \begin{array}{l|l} 9 & 2 \\ 8 & 4 \\ =1 & 4 \\ & =0 \end{array} \quad \begin{array}{l|l} 4 & 2 \\ 4 & 2 \\ =0 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 2 & 2 \\ 2 & 1 \\ =0 & \end{array}$$

$$\begin{array}{l|l} 85 & 2 \\ 8 & 42 \\ =5 & 2 \\ 4 & =2 \\ =1 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 42 & 2 \\ 42 & 21 \\ =0 & 2 \\ & =1 \\ & 0 \\ & =1 \end{array} \quad \begin{array}{l|l} 21 & 2 \\ 10 & 10 \\ =1 & 10 \\ & =0 \end{array} \quad \begin{array}{l|l} 10 & 2 \\ 5 & 2 \\ =0 & 2 \\ & =1 \end{array} \quad \begin{array}{l|l} 5 & 2 \\ 4 & 2 \\ =1 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 2 & 2 \\ 2 & 1 \\ =0 & \end{array}$$

$$\begin{array}{l|l} 97 & 2 \\ 8 & 48 \\ =1 & 2 \\ 17 & =8 \\ 16 & 8 \\ =1 & =0 \end{array} \quad \begin{array}{l|l} 48 & 2 \\ 48 & 24 \\ =0 & 2 \\ & =4 \\ & 4 \\ & =0 \end{array} \quad \begin{array}{l|l} 24 & 2 \\ 24 & 12 \\ =0 & 12 \\ & =0 \end{array} \quad \begin{array}{l|l} 12 & 2 \\ 6 & 6 \\ =0 & 6 \\ & =0 \end{array} \quad \begin{array}{l|l} 6 & 2 \\ 3 & 2 \\ =0 & 1 \end{array}$$

5. a) $2010 = 11111011010_{(2)} =$

$= 1\cdot 2^{10} + 1\cdot 2^{9} + 1\cdot 2^{8} + 1\cdot 2^{7} +$

$+ 1\cdot 2^{6} + 1\cdot 2^{4} + 1\cdot 2^{3} + 1\cdot 2 =$

$= 2^{10} + 2^{9} + 2^{8} + 2^{7} + 2^{6} + 2^{4} + 2^{3} + 2$

$$\begin{array}{l|l} 2010 & 2 \\ 2 & 1005 \\ =0 & 10 \\ 0 & =0 \\ =1 & 0 \\ 0 & =5 \\ 10 & 4 \\ 10 & =1 \\ =0 & \end{array} \quad \begin{array}{l|l} 1005 & 2 \\ 10 & 502 \\ =0 & 4 \\ & 10 \\ & 10 \\ & =2 \\ & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 502 & 2 \\ 4 & 251 \\ 10 & 2 \\ 10 & =5 \\ =2 & 4 \\ 2 & 11 \\ =0 & 10 \\ & =1 \end{array} \quad \begin{array}{l|l} 251 & 2 \\ 2 & 125 \\ =5 & 12 \\ 4 & =5 \\ 11 & 4 \\ 10 & =1 \\ =1 & \end{array} \quad \begin{array}{l|l} 125 & 2 \\ 12 & 62 \\ =5 & 6 \\ 4 & =2 \\ =1 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 62 & 2 \\ 6 & 31 \\ =2 & 2 \\ 2 & 11 \\ =0 & 10 \\ & =1 \end{array} \quad \begin{array}{l|l} 31 & 2 \\ 2 & 15 \\ 11 & 14 \\ 10 & =1 \\ =1 & \end{array} \quad \begin{array}{l|l} 15 & 2 \\ 14 & 7 \\ =1 & 6 \\ & =1 \end{array} \quad \begin{array}{l|l} 7 & 2 \\ 6 & 3 \\ =1 & 2 \\ & =1 \end{array}$$

b) $1999 = 11111001111_{(2)} =$

$= 1\cdot 2^{10} + 1\cdot 2^{9} + 1\cdot 2^{8} + 1\cdot 2^{7} + 1\cdot 2^{6} +$

$+ 1\cdot 2^{3} + 1\cdot 2^{2} + 1\cdot 2 + 1 =$

$= 2^{10} + 2^{9} + 2^{8} + 2^{7} + 2^{6} + 2^{3} + 2^{2} + 2^{1} + 2^{0}$

$$\begin{array}{l|l} 1999 & 2 \\ 18 & 999 \\ =19 & 8 \\ 18 & 19 \\ =19 & 18 \\ 18 & =19 \\ =1 & 18 \\ & =1 \end{array} \quad \begin{array}{l|l} 999 & 2 \\ 8 & 499 \\ 19 & 4 \\ 18 & =9 \\ =19 & 8 \\ 18 & 19 \\ =1 & 18 \\ & =1 \end{array} \quad \begin{array}{l|l} 499 & 2 \\ 4 & 249 \\ =9 & 2 \\ & =4 \\ & 4 \\ & =9 \\ & 8 \\ & =1 \end{array} \quad \begin{array}{l|l} 249 & 2 \\ 2 & 124 \\ =4 & 12 \\ 4 & =4 \\ =9 & 4 \\ 8 & =0 \\ =1 & \end{array} \quad \begin{array}{l|l} 124 & 2 \\ 12 & 62 \\ =4 & 6 \\ 4 & =2 \\ =0 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 62 & 2 \\ 6 & 31 \\ =2 & 2 \\ 2 & 11 \\ =0 & 10 \\ & =1 \end{array} \quad \begin{array}{l|l} 31 & 2 \\ 2 & 15 \\ 11 & 14 \\ 10 & =1 \\ =1 & \end{array} \quad \begin{array}{l|l} 15 & 2 \\ 14 & 7 \\ =1 & 6 \\ & =1 \end{array} \quad \begin{array}{l|l} 7 & 2 \\ 6 & 3 \\ =1 & 2 \\ & =1 \end{array}$$

6. $a = 10010_{(2)}$

$b = 1\cdot 2^{4} + 1\cdot 2 = 2^{4} + 2$

$c = 1\cdot 10 + 8\cdot 10^{0}$

$d = 1\cdot 2^{3} + 1\cdot 2 = 8 + 2 = 10$

$e = 2^{3} + 2$

$f = 1\cdot 10$

$g = 2^{5} + 2^{2} + 1 = 32 + 4 + 1 = 37$

$h = 100101_{(2)}$

$$\begin{array}{l|l} 18 & 2 \\ 18 & 9 \\ =0 & 8 \\ & =1 \\ & 4 \\ & =0 \\ & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 9 & 2 \\ 4 & 2 \\ =1 & 2 \\ & =0 \end{array} \quad \begin{array}{l|l} 4 & 2 \\ 2 & 2 \\ =0 & 2 \\ & =1 \end{array} \quad \begin{array}{l|l} 2 & 2 \\ 2 & 1 \\ =0 & \end{array}$$

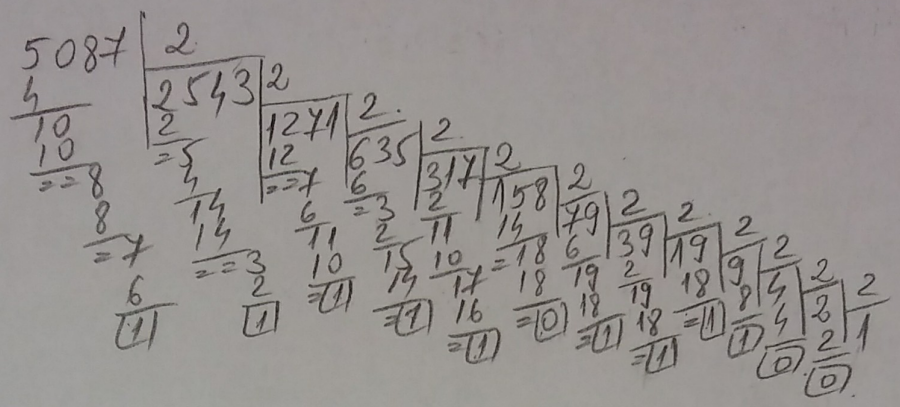
$i = 3 \cdot 10 + 7 \cdot 10^0$

$j = 5 \cdot 10^3 + 8 \cdot 10 + 7 = 5 \cdot 1000 + 80 + 7 = 5087$

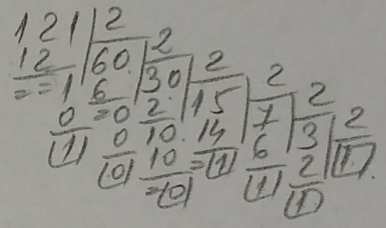
$k = 1001110111111_{(2)}$

$l = 1 \cdot 2^{12} + 1 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2 + 1 =$

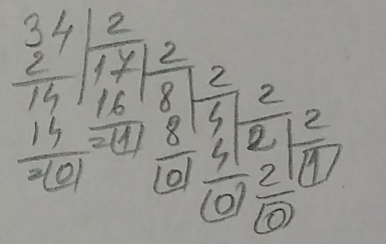
$= 2^{12} + 2^9 + 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^2 + 2 + 1$



7. a) $121_{(10)} = x_{(2)} \Rightarrow x = 1111001$



b) $34_{(10)} = x_{(2)} \Rightarrow x = 100010$



c) $11011_{(2)} = x_{(10)} \Rightarrow$

$\Rightarrow x = 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2 + 1 = 16 + 8 + 2 + 1 \Rightarrow$

$\Rightarrow x = 27$

d) $101011_{(2)} = x_{(10)} \Rightarrow x = 1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2 + 1 = 32 + 8 + 2 + 1 = 43$

8. a) $29_{(10)} = 10001_{(2)}$ (F)

$10001_{(2)} = 1 \cdot 2^4 + 1 = 16 + 1 = 17$

b) $111_{(2)} = 1 \cdot 2^2 + 1 \cdot 2 + 1 = 4 + 2 + 1 = 7 \Rightarrow 111_{(2)}$ nu este pătratul unui număr natural (A)

c) $1101_{(2)} > 3^4 : 3^2$ (A)

$1101_{(2)} = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 = 8 + 4 + 1 = 13$

$3^4 : 3^2 = 3^{4-2} = 3^2 = 9.$

d) $11001_{(2)} = 1 \cdot 2^4 + 1 \cdot 2^3 + 1 = 16 + 8 + 1 = 25 = 5^2 \Rightarrow 11001_{(2)}$ este pătratul unui număr natural (A)