Lecția 9

Puterea cu exponent natural a unei fracții ordinare

1. a) $\left(\frac{1}{2}\right)^7 = \frac{1^7}{2^7} = \frac{1}{128}$

b) $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

c) $\left(\frac{4}{9}\right)^3 = \frac{4^3}{9^3} = \frac{64}{729}$

d) $\left(\frac{11}{7}\right)^3 = \frac{11^3}{7^3} = \frac{1331}{343}$

e) $\left(\frac{5}{4}\right)^4 = \frac{5^4}{4^4} = \frac{625}{256}$

f) $\left(\frac{3}{3}\right)^{2017} = 1^{2017} = 1$

g) $\left(\frac{19}{43}\right)^1 = \frac{19^1}{43^1} = \frac{19}{43}$

h) $\left(\frac{2011}{2017}\right)^0 = 1$, deoarece $a^0 = 1$.

2. a) $\left(\frac{2}{3}\right)^5 \cdot \left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^{5+7} = \left(\frac{2}{3}\right)^{12}$

b) $\left(\frac{3}{10}\right)^3 \cdot \left(\frac{3}{10}\right)^{10} = \left(\frac{3}{10}\right)^{3+10} = \left(\frac{3}{10}\right)^{13}$

c) $\left(\frac{11}{5}\right)^4 \cdot \left(\frac{11}{5}\right)^6 = \left(\frac{11}{5}\right)^{4+6} = \left(\frac{11}{5}\right)^{10}$

d) $\left(\frac{3}{4}\right)^{11} : \left(\frac{3}{4}\right)^4 = \left(\frac{3}{4}\right)^{11-4} = \left(\frac{3}{4}\right)^7$

e) $\left(\frac{8}{3}\right)^9 : \left(\frac{8}{3}\right)^8 = \left(\frac{8}{3}\right)^{9-8} = \left(\frac{8}{3}\right)^1 = \frac{8}{3}$

f) $\left(\frac{13}{17}\right)^{22} : \left(\frac{13}{17}\right)^{20} = \left(\frac{13}{17}\right)^{22-20} = \left(\frac{13}{17}\right)^2$

g) $\left[\left(\frac{2}{15}\right)^5\right]^{11} = \left(\frac{2}{5}\right)^{5 \cdot 11} = \left(\frac{2}{5}\right)^{55}$

h) $\left[\left(\frac{14}{27}\right)^6\right]^4 = \left(\frac{14}{27}\right)^{6 \cdot 4} = \left(\frac{14}{27}\right)^{24}$

i) $\left[\left(\frac{3}{100}\right)^0\right]^{2017} = \left(\frac{3}{100}\right)^{0 \cdot 2017} = \left(\frac{3}{100}\right)^0 = 1$

3. a) $\left(\frac{35}{95}\right)^6 \cdot \left(\frac{38}{14}\right)^6 = \left(\frac{\cancel{35}^{5}}{\cancel{95}_{19}} \cdot \frac{\cancel{38}^{2}}{\cancel{14}_{2}}\right)^6 = \left(\frac{1}{1} \cdot \frac{1}{1}\right)^6 = 1^6 = 1$

b) $\left(\frac{33}{50}\right)^9 \cdot \left(\frac{25}{22}\right)^9 = \left(\frac{\cancel{33}^{3}}{\cancel{50}_{2}} \cdot \frac{\cancel{25}^{1}}{\cancel{22}_{2}}\right)^9 = \left(\frac{3}{2} \cdot \frac{1}{2}\right)^9 = \left(\frac{3}{4}\right)^9$

c) $\left(\frac{3}{7}\right)^4 \cdot \left(\frac{49}{36}\right)^2 = \left(\frac{3}{7}\right)^4 \cdot \left(\frac{7^2}{6^2}\right)^2 = \left(\frac{3}{7}\right)^4 \cdot \left[\left(\frac{7}{6}\right)^2\right]^2 = \left(\frac{3}{7}\right)^4 \cdot \left(\frac{7}{6}\right)^{2\cdot 2} =$

$= \left(\frac{3}{7}\right)^4 \cdot \left(\frac{7}{6}\right)^4 = \left(\frac{\cancel{3}^{1}}{\cancel{7}_{1}} \cdot \frac{\cancel{7}^{1}}{\cancel{6}_{2}}\right)^4 = \left(\frac{1}{1} \cdot \frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4$

sau

$\left(\frac{3}{7}\right)^4 \cdot \left(\frac{49}{36}\right)^2 = \left(\frac{3}{7}\right)^{2\cdot 2} \cdot \left(\frac{49}{36}\right)^2 = \left[\left(\frac{3}{7}\right)^2\right]^2 \cdot \left(\frac{49}{36}\right)^2 =$

$= \left(\frac{3^2}{7^2}\right)^2 \cdot \left(\frac{49}{36}\right)^2 = \left(\frac{9}{49}\right)^2 \cdot \left(\frac{49}{36}\right)^2 = \left(\frac{\cancel{9}^{1}}{\cancel{49}_{1}} \cdot \frac{\cancel{49}^{1}}{\cancel{36}_{4}}\right)^2 =$

$= \left(\frac{1}{1} \cdot \frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 = \left(\frac{1}{2^2}\right)^2 = \left[\left(\frac{1}{2}\right)^2\right]^2 = \left(\frac{1}{2}\right)^{2\cdot 2} = \left(\frac{1}{2}\right)^4$

d) $\left(\frac{7}{17}\right)^5 : \left(\frac{21}{17}\right)^5 = \left(\frac{7}{17} : \frac{21}{17}\right)^5 = \left(\frac{\cancel{7}^{1}}{\cancel{17}_{1}} \cdot \frac{\cancel{17}^{1}}{\cancel{21}_{3}}\right)^5 = \left(\frac{1}{1} \cdot \frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^5$

e) $\left(\frac{13}{8}\right)^6 : \left(\frac{39}{32}\right)^6 = \left(\frac{13}{8} : \frac{39}{32}\right)^6 = \left(\frac{\cancel{13}^{1}}{\cancel{8}_{1}} \cdot \frac{\cancel{32}^{4}}{\cancel{39}_{3}}\right)^6 = \left(\frac{1}{1} \cdot \frac{4}{3}\right)^6 = \left(\frac{4}{3}\right)^6$

f) $\left(4\frac{1}{6}\right)^{12} : \left(11\frac{1}{9}\right)^6 = \left(\frac{4\cdot 6+1}{6}\right)^{12} : \left(\frac{11\cdot 9+1}{9}\right)^6 = \left(\frac{25}{6}\right)^{12} : \left(\frac{100}{9}\right)^6 =$

$= \left(\frac{25}{6}\right)^{12} : \left(\frac{10^2}{3^2}\right)^6 = \left(\frac{25}{6}\right)^{12} : \left[\left(\frac{10}{3}\right)^2\right]^6 = \left(\frac{25}{6}\right)^{12} : \left(\frac{10}{3}\right)^{12} = \left(\frac{25}{6} \cdot \frac{10}{3}\right)^{12} =$

$= \left(\frac{\cancel{25}^{5}}{\cancel{6}_{2}} \cdot \frac{\cancel{3}^{1}}{\cancel{10}_{2}}\right)^{12} = \left(\frac{5}{2} \cdot \frac{1}{2}\right)^{12} = \left(\frac{5}{4}\right)^{12}$

4. a) $\left(\frac{1}{256}\right) = \frac{1}{2^8} = \frac{1^8}{2^8} = \left(\frac{1}{2}\right)^8$

256	2
128	2
64	2
32	2
16	2
8	2
4	2
2	2
1	

b) $\frac{16}{81} = \frac{2^4}{3^4} = \left(\frac{2}{3}\right)^4$

c) $\frac{125}{216} = \frac{5^3}{6^3} = \left(\frac{5}{6}\right)^3$

216	2
108	2
54	2
27	3
9	3
3	3
1	

$216 = 2^3 \cdot 3^3 = (2 \cdot 3)^3 = 6^3$

$125 = 5 \cdot 5 \cdot 5 = 5^3$

5. a) $\frac{1}{64} = \frac{1}{2^6} = \frac{1^6}{2^6} = \left(\frac{1}{2}\right)^6$

$64 = 8 \cdot 8 = 2^3 \cdot 2^3 = 2^{3+3} = 2^6$

b) $\frac{32}{243} = \frac{2^5}{3^5} = \left(\frac{2}{3}\right)^5$

243	3
81	3
27	3
9	3
3	3
1	

$32 = 2^5$

$243 = 3^5$

c) $\frac{343}{216} = \frac{7^3}{6^3} = \left(\frac{7}{6}\right)^3$

343	7
49	7
7	7
1	

$343 = 7 \cdot 7 \cdot 7 = 7^3$

$216 = 2^3 \cdot 3^3 = (2 \cdot 3)^3 = 6^3$