

Limitest

Unitatea III
Lectia 3.

1. a) $137 = \text{nr. prim?}$

$137 \not\div 2$ (deoarece 137 nu se termină în cifră pară)

$137 \not\div 3$ (deoarece $(1+3+7) \not\div 3 \Rightarrow 11 \not\div 3$)

$137 \not\div 5$ (deoarece 137 nu se termină cu cifrele 0 și 5)

$137 : 7 = 19 \text{ rest } 4$

$$\begin{array}{r} 137 \overline{)7} \\ \underline{7} \\ 67 \\ \underline{63} \\ 4 \neq 0 \end{array} > 7$$

$137 : 11 = 12 \text{ rest } 5$

$$\begin{array}{r} 137 \overline{)11} \\ \underline{11} \\ 27 \\ \underline{22} \\ 5 \neq 0 \end{array} > 11$$

$137 : 13 = 10 \text{ rest } 7$

$$\begin{array}{r} 137 \overline{)13} \\ \underline{13} \\ 7 \\ \underline{0} \\ 7 \neq 0 \end{array} < 13$$

$\boxed{10} < 13 \Rightarrow 137$ este nr. prim

b) $187 = \text{nr. compus}$

$187 \not\div 2$

$187 \not\div 3$

$187 \not\div 5$

$187 : 7 = 26 \text{ rest } 5$

$$\begin{array}{r} 187 \overline{)7} \\ \underline{14} \\ 47 \\ \underline{42} \\ 5 \neq 0 \end{array} > 7$$

$187 : 11 = 17 \text{ rest } 0 \Rightarrow$

$$\begin{array}{r} 187 \overline{)11} \\ \underline{11} \\ 77 \\ \underline{77} \\ 0 \end{array} =$$

$\Rightarrow 187$ este nr. compus

$$187 = 11 \cdot 17$$

c) $263 = \text{nr. prim}$

$$263 \not\div 2$$

$$263 \not\div 3$$

$$263 \not\div 5$$

$$263 : 7 = 37 \text{ rest } 4$$

$$263 : 11 = 23 \text{ rest } 10$$

$$263 : 13 = 20 \text{ rest } 3$$

$$263 : 17 = 15 \text{ rest } 8$$

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$15 < 17 \Rightarrow 263 \text{ este nr. prim}$

$$\begin{array}{r} 263 \overline{) 7} \\ \underline{21} \\ =53 \\ \underline{49} \\ =4 \neq 0 \end{array} > 7$$

$$\begin{array}{r} 263 \overline{) 11} \\ \underline{22} \\ =43 \\ \underline{33} \\ =10 \neq 0 \end{array} > 11$$

$$\begin{array}{r} 263 \overline{) 13} \\ \underline{26} \\ =3 \\ \underline{0} \\ =3 \neq 0 \end{array} > 13$$

$$\begin{array}{r} 263 \overline{) 17} \\ \underline{17} \\ =93 \\ \underline{85} \\ =8 \neq 0 \end{array} < 17$$

2. a) $\overline{abc} = \text{nr. prim}$

$$a \cdot b \cdot c = 3$$

$$1 \cdot 1 \cdot 3 = 3 \Rightarrow 113$$

$$1 \cdot 3 \cdot 1 = 3 \Rightarrow 131$$

$$3 \cdot 1 \cdot 1 = 3 \Rightarrow 311$$

$$113 \not\div 2$$

$$113 \not\div 3$$

$$113 \not\div 5$$

$$113 : 7 = 16 \text{ rest } 1$$

$$113 : 11 = 10 \text{ rest } 3$$

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$10 < 11 \Rightarrow 113 \text{ nr. prim}$

$$\begin{array}{r} 113 \overline{) 7} \\ \underline{7} \\ =43 \\ \underline{42} \\ =1 \neq 0 \end{array} > 7$$

$$\begin{array}{r} 113 \overline{) 11} \\ \underline{11} \\ =3 \\ \underline{0} \\ =3 \neq 0 \end{array} < 11$$

$$131 \not\div 2$$

$$131 \not\div 3$$

$$131 \not\div 5$$

$$131 : 7 = 18 \text{ rest } 5$$

$$131 : 11 = 11 \text{ rest } 10$$

$$131 : 13 = 10 \text{ rest } 1$$

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$10 < 13 \Rightarrow 131 \text{ nr. prim}$

$$\begin{array}{r} 131 \overline{) 7} \\ \underline{7} \\ =61 \\ \underline{56} \\ =5 \neq 0 \end{array} > 7$$

$$\begin{array}{r} 131 \overline{) 11} \\ \underline{11} \\ =21 \\ \underline{11} \\ =10 \neq 0 \end{array} = 11$$
$$\begin{array}{r} 131 \overline{) 13} \\ \underline{13} \\ =22 \\ \underline{21} \\ =1 \neq 0 \end{array} < 13$$

$$311 \neq 2$$

$$311 \neq 3$$

$$311 \neq 5$$

$$311 : 7 = 44 \text{ rest } 3$$

$$311 : 11 = 28 \text{ rest } 3$$

$$311 : 13 = 23 \text{ rest } 12$$

$$311 : 17 = 18 \text{ rest } 5$$

$$311 : 19 = 16 \text{ rest } 7$$

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$16 < 19 \Rightarrow 311$ nr. prim

$$\begin{array}{r} 311 \overline{) 7} \\ \underline{28} \\ 28 \\ \underline{28} \\ 0 \end{array} > 7$$

$$\begin{array}{r} 311 \overline{) 11} \\ \underline{22} \\ 91 \\ \underline{88} \\ 3 \end{array} > 11$$

$$\begin{array}{r} 311 \overline{) 13} \\ \underline{26} \\ 51 \\ \underline{39} \\ 12 \end{array} > 13$$

$$\begin{array}{r} 311 \overline{) 17} \\ \underline{17} \\ 141 \\ \underline{136} \\ 5 \end{array} > 17$$

$$\begin{array}{r} 311 \overline{) 19} \\ \underline{19} \\ 121 \\ \underline{114} \\ 7 \end{array} < 19$$

Deci, avem 3 numere prime de trei cifre, cu produsul egal cu 5.

b) $a \cdot b \cdot c = \text{nr. compus}$

$$a \cdot b \cdot c = 5$$

$$1 \cdot 1 \cdot 5 = 5 \Rightarrow 115$$

$$1 \cdot 5 \cdot 1 = 5 \Rightarrow 151$$

$$5 \cdot 1 \cdot 1 = 5 \Rightarrow 511$$

$$115 : 5 \Rightarrow 115 \text{ nr. compus}$$

$$115 = 5 \cdot 23$$

$$151 \neq 2$$

$$151 \neq 3$$

$$151 \neq 5$$

$$151 : 7 = 21 \text{ rest } 4$$

$$151 : 11 = 13 \text{ rest } 8$$

$$151 : 13 = 11 \text{ rest } 8$$

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$11 < 13 \Rightarrow 151$ nr. prim

$$\begin{array}{r} 151 \overline{) 7} \\ \underline{14} \\ 11 \\ \underline{7} \\ 4 \end{array} > 7$$

$$\begin{array}{r} 151 \overline{) 11} \\ \underline{11} \\ 41 \\ \underline{33} \\ 8 \end{array} > 11$$

$$\begin{array}{r} 151 \overline{) 13} \\ \underline{13} \\ 21 \\ \underline{21} \\ 13 \\ \underline{13} \\ 0 \end{array} < 13$$

$$511 \div 2$$

$$511 \div 3$$

$$511 \div 5$$

$$\begin{array}{r|l} 511 & 7 \\ \underline{49} & 73 \\ \hline & 21 \\ \underline{21} & \\ \hline & 0 \end{array}$$

$$511 : 7 = 73 \text{ rest } 0 \Rightarrow 511 \text{ este nr. compus}$$

$$511 = 7 \cdot 73$$

Deci, avem 2 numere compuse de trei cifre, cu produsul cifrelor egal cu 5.

3. $a, b, c = ?$ nr. prime

$$2a + 7b + 6c = 78$$

$$2a : 2$$

$$6c : 2$$

$$78 : 2$$

$$\Rightarrow 7b = 78 - 2a - 6c \quad \Bigg| \Rightarrow 7b : 2 \Rightarrow$$
$$(78 - 2a - 6c) : 2$$

$$\Rightarrow b : 2 \Rightarrow b = 2$$

$$2a + 7 \cdot 2 + 6c = 78$$

$$2a + 14 + 6c = 78 \quad | : 2$$

$$a + 7 + 3c = 39$$

$$a + 3c = 39 - 7$$

$$a + 3c = 32 \Rightarrow 3c = 32 - a \quad \left| \begin{array}{l} 3c : 3 \\ \hline \Rightarrow (32 - a) : 3 \Rightarrow (32 - a) \in \mathcal{M}_3 \end{array} \right.$$

$$32 - a = 33 \Rightarrow a = 32 - 33 \text{ imposibil}$$

$$32 - a = 30 \Rightarrow a = 32 - 30 \Rightarrow a = 2$$

$$3c = 32 - 2 = 30 \Rightarrow c = 10 \neq \text{prim}$$

$$32 - a = 27 \Rightarrow a = 32 - 27 \Rightarrow a = 5$$

$$3c = 32 - 5 \Rightarrow 3c = 27 \Rightarrow c = 9 \neq \text{prim}$$

$$32 - a = 24 \Rightarrow a = 32 - 24 \Rightarrow a = 7$$

$$3c = 32 - 24 \Rightarrow 3c = 8 \Rightarrow \text{imposibil}$$

$$32 - a = 21 \Rightarrow a = 32 - 21 \Rightarrow \boxed{a = 11}$$

$$3c = 32 - 11 \Rightarrow 3c = 21 \Rightarrow \boxed{c = 7}$$

$$32 - a = 18 \Rightarrow a = 32 - 18 \Rightarrow a = 14 \neq \text{prim}$$

$$32 - a = 15 \Rightarrow a = 32 - 15 \Rightarrow \boxed{a = 17}$$

$$3c = 15 \Rightarrow c = 15 : 3 \Rightarrow \boxed{c = 5}$$

$$32 - a = 12 \Rightarrow a = 32 - 12 \Rightarrow a = 20 \neq \text{prim}$$

$$32 - a = 9 \Rightarrow \boxed{a = 23}$$

$$3c = 32 - 23 \Rightarrow 3c = 9 \Rightarrow \boxed{c = 3}$$

$$32 - a = 6 \Rightarrow a = 32 - 6 \Rightarrow a = 26 \neq \text{prim}$$

$$32 - a = 3 \Rightarrow a = 32 - 3 \Rightarrow \boxed{a = 29}$$

$$3c = 32 - 29 \Rightarrow 3c = 3 \Rightarrow \boxed{c = 1} \text{ nu este considerat nr. prim}$$

Deci, avem numerele: $a = 11, b = 2, c = 7$
sau $a = 17, b = 2, c = 5$
sau $a = 23, b = 2, c = 3$

4. $m, n, p = ?$ prime

$$m \cdot n = 28 - p$$

$p, m, n \neq 1$, deoarece 1 nu este considerat nr. prim

Pentru $p = 2$

$$m \cdot n = 28 - 2 \Rightarrow m \cdot n = 26$$

$$2 \cdot 13 = 26$$

$$\left. \begin{array}{l} m \cdot n = 26 \\ 2 \cdot 13 = 26 \end{array} \right\} \Rightarrow m = 2, n = 13 \text{ sau } m = 13, n = 2$$

$p = 3$

$$m \cdot n = 28 - 3 \Rightarrow m \cdot n = 25$$

$$5 \cdot 5 = 25$$

$$\left. \begin{array}{l} m \cdot n = 25 \\ 5 \cdot 5 = 25 \end{array} \right\} \Rightarrow m = n = 5$$

$p = 5$

$$m \cdot n = 28 - 5 \Rightarrow m \cdot n = 23$$

$$1 \cdot 23 = 23$$

$$23 \cdot 1 = 23$$

$$\left. \begin{array}{l} m \cdot n = 23 \\ 1 \cdot 23 = 23 \\ 23 \cdot 1 = 23 \end{array} \right\} \Rightarrow \text{nu convine, deoarece } 1 \text{ nu este nr. prim}$$

$p = 7$

$$m \cdot n = 28 - 7$$

$$m \cdot n = 21$$

$$3 \cdot 7 = 21$$

$$\Rightarrow m = 3, n = 7$$

$$7 \cdot 3 = 21$$

$$\Rightarrow m = 7, n = 3$$

$p = 11$

$$m \cdot n = 28 - 11$$

$$m \cdot n = 17 \Rightarrow 1 \cdot 17 = 17 \text{ sau } 17 \cdot 1 = 17 \text{ (nu convine)}$$

$p = 13$

$$m \cdot n = 28 - 13 \Rightarrow m \cdot n = 15$$

$$3 \cdot 5 = 15 \Rightarrow m = 3, n = 5$$

$$5 \cdot 3 = 15 \Rightarrow m = 5, n = 3$$

$p = 17$

$$m \cdot n = 28 - 17 \Rightarrow m \cdot n = 11 \Rightarrow 1 \cdot 11 = 11 \text{ sau } 11 \cdot 1 = 11 \text{ (nu convine)}$$

$$p = 19 \Rightarrow m \cdot n = 28 - 19 \Rightarrow m \cdot n = 9 \Rightarrow m = n = 3$$

$$p = 23 \Rightarrow m \cdot n = 28 - 23 \Rightarrow m \cdot n = 5 \Rightarrow 1 \cdot 5 = 5 \text{ nu convine}$$

Deci, avem: $(2, 13, 2); (13, 2, 2); (5, 5, 3); (3, 7, 7); (7, 3, 7); (3, 5, 13); (5, 3, 13); (3, 3, 19)$