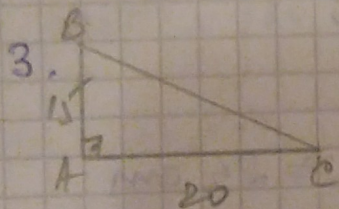


3.6. Raporturi constante în triunghiul dreptunghic: sin, cos, tg, ctg

Aplicații

- a) $\sin^2 m + \cos^2 m = 1$
b) $\sin(90 - m) = \cos m$
c) $\text{ctg}(90 - m) = \text{tg } m$
d) $\frac{\sin m}{\cos m} = \text{tg } m$
e) $\frac{\cos m}{\sin m} = \text{ctg } m$

$$\begin{aligned} 2. \quad & \sin 30^\circ \cdot \cos 30^\circ + \text{tg } 60^\circ \cdot \text{ctg } 60^\circ + \sin 45^\circ \cdot \cos 45^\circ - \\ & - \sin 60^\circ \cdot \cos 60^\circ = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \sqrt{3} \cdot \frac{\sqrt{3}}{3} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \\ & = \frac{3}{4} + \frac{2}{4} = \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1 \end{aligned}$$



$$m(\hat{A}) = 90^\circ$$

$$AB = 15 \text{ cm}$$

$$AC = 20 \text{ cm}$$

$$a) \sin \hat{B} = \frac{AC}{BC} = \frac{20}{BC}$$

$$\Delta ABC \text{ dr.} \xrightarrow{\text{Th.P.}} BC^2 = AC^2 + AB^2$$

$$BC^2 = 400 + 225 = 625$$

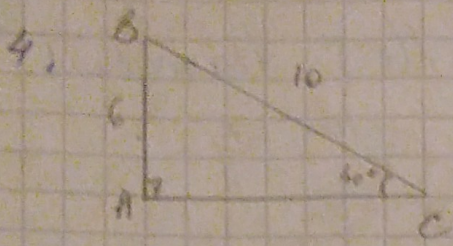
$$BC = \sqrt{625} = 25 \text{ cm}$$

$$\sin \hat{B} = \frac{20^{15}}{25} = \frac{4}{5}$$

$$b) \cos \hat{B} = \frac{AB}{BC} = \frac{15^{15}}{25} = \frac{3}{5}$$

$$c) \text{tg } \hat{B} = \frac{AC}{AB} = \frac{20^{15}}{15} = \frac{4}{3}$$

$$d) \text{ctg } \hat{B} = \frac{AB}{AC} = \frac{15^{15}}{20} = \frac{3}{4}$$



ΔABC

$$BC = 10 \text{ cm}$$

$$m(\hat{C}) = 40^\circ$$

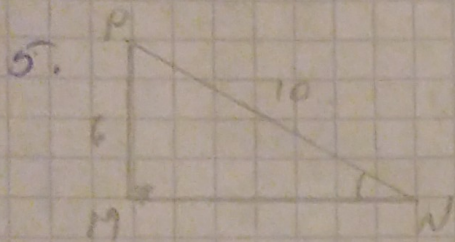
$$\sin(\hat{C}) = \frac{AB}{10} \Leftrightarrow \sin 40^\circ = \frac{AB}{10} \Rightarrow AB = 10 \cdot \sin 40^\circ =$$

$$= 10 \cdot 0,6 = 6 \text{ cm}$$

$$\sin 40^\circ = 0,642 \approx 0,6$$

$$AC^2 = BC^2 - AB^2 = 100 - 36 = 64$$

$$AC = \sqrt{64} = 8 \text{ cm}$$



ΔPNM - dreptunghi

$$m(\hat{M}) = 90^\circ$$

$$PM = 6 \text{ cm}$$

$$\sin \hat{N} = 0,6 \Rightarrow \hat{N} = 40^\circ$$

$$\sin \hat{N} = 0,6$$

$$\sin \hat{N} = \frac{PM}{PN} = 0,6 \Leftrightarrow \frac{6}{PN} = 0,6 \Leftrightarrow PN = \frac{6}{0,6} = \frac{6 \cdot 10}{6} = 10$$

Th P.

$$\Rightarrow MN^2 = PN^2 - PM^2 = 100 - 36 = 64$$

$$MN = \sqrt{64} = 8 \text{ cm}$$

$$P_{\Delta PMN} = PM + MN + NP = 6 + 8 + 10 = 24 \text{ cm}$$

6. $x < 90^\circ$, $\sin x = \frac{3}{5}$

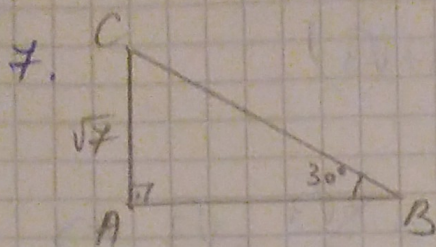
a) $\cos^2 x + \sin^2 x = 1 \Leftrightarrow \cos^2 x + \left(\frac{3}{5}\right)^2 = 1 \Leftrightarrow$

$$\Leftrightarrow \cos^2 x = 1 - \frac{9}{25} \Leftrightarrow \cos^2 x = \frac{16}{25} \Rightarrow$$

$$\Rightarrow \cos x = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

b) $\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{3}{4} \cdot \frac{5}{4} = \frac{3}{4}$

c) $\operatorname{ctg} x = \frac{\cos x}{\sin x} = \frac{4}{3} \cdot \frac{5}{3} = \frac{4}{3}$



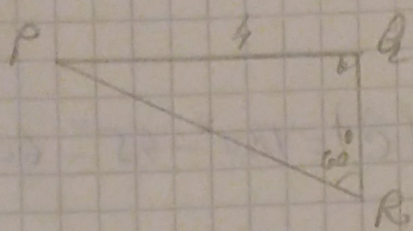
$$\sin 30^\circ = \frac{AC}{BC} = \frac{\sqrt{7}}{BC} \Rightarrow$$

$$\Rightarrow \frac{1}{2} = \frac{\sqrt{7}}{BC} \Rightarrow BC = 2\sqrt{7}$$

$$\cos 30^\circ = \frac{AB}{BC} = \frac{\sqrt{3}}{2} \Rightarrow \frac{AB}{2\sqrt{7}} = \frac{\sqrt{3}}{2} \Rightarrow$$

$$\Rightarrow AB = \frac{2\sqrt{21}}{2} = \sqrt{21}$$

$$m(\hat{C}) = 180^\circ - m(\hat{A}) - m(\hat{B}) = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$



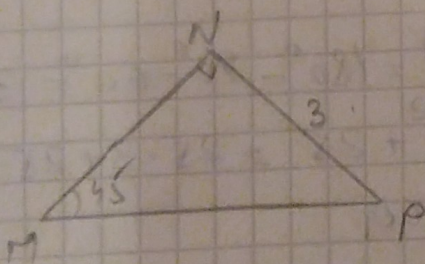
$$\sin 60^\circ = \frac{PQ}{PR} = \frac{\sqrt{3}}{2} \Rightarrow \frac{4}{PR} = \frac{\sqrt{3}}{2} \Rightarrow$$

$$\Rightarrow PR = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$$\cos 60^\circ = \frac{QR}{PR} = \frac{1}{2} \Rightarrow \frac{QR}{\frac{8\sqrt{3}}{3}} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow QR = \frac{8\sqrt{3}}{3} \cdot \frac{1}{2} = \frac{4\sqrt{3}}{3}$$

$$m(\hat{P}) = 180^\circ - m(\hat{Q}) - m(\hat{R}) = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$



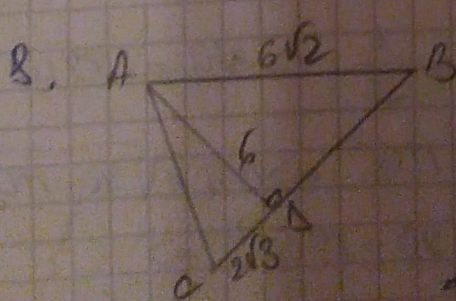
$$\sin 45^\circ = \frac{PN}{MP} = \frac{\sqrt{2}}{2} \Rightarrow \frac{3}{MP} = \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow MP = \frac{6}{\sqrt{2}} = \frac{3\sqrt{2}}{1} = 3\sqrt{2}$$

$$\cos 45^\circ = \frac{MN}{MP} = \frac{\sqrt{2}}{2} \Rightarrow \frac{MN}{3\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow MN = \frac{3\sqrt{2} \cdot \sqrt{2}}{2} = \frac{3 \cdot 2}{2} = 3$$

$$m(\hat{P}) = 180^\circ - 90^\circ - 45^\circ = 45^\circ$$



$$\Delta ADB - dr \quad \sin B = \frac{AD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = 45^\circ$$

$$\text{Dacă } m(\hat{B}) = 45^\circ \Rightarrow m(\Delta ADB) = 45^\circ \Rightarrow$$

$$\Rightarrow \Delta ADB - \text{dreptunghiuc isoscel} \Rightarrow$$

$$\Rightarrow AD = DB = 6$$

$$BC = BA + AC = 6 + 2\sqrt{3} = 2(3 + \sqrt{3})$$

$\triangle ADC$ - dreptunghi

$$\operatorname{tg} \hat{C} = \frac{AD}{CA} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3} \Rightarrow$$

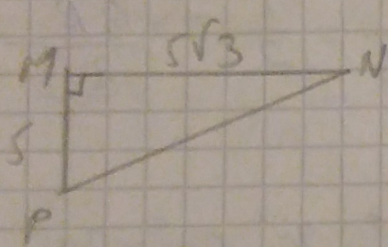
$$\Rightarrow \hat{C} = 60^\circ$$

$$\text{Th. P. } AC^2 = AD^2 + CA^2 = 36 + (2\sqrt{3})^2 = 36 + 12 = 48$$

$$AC = \sqrt{48} = 4\sqrt{3}$$

$\triangle ABC$

$$m(\hat{A}) = 180^\circ - m(\hat{B}) - m(\hat{C}) = 180^\circ - 45^\circ - 60^\circ = 75^\circ$$



$\triangle PMN$ - dreptunghi

$$\operatorname{tg}(\hat{P}) = \frac{MN}{MP} = \frac{5\sqrt{3}}{5} = \sqrt{3} \Rightarrow$$

$$\Rightarrow \hat{P} = 60^\circ$$

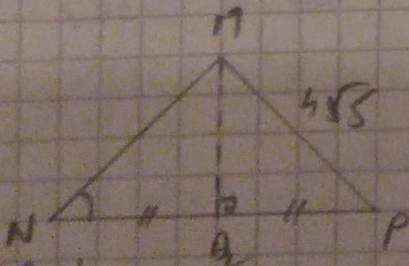
$$m(\hat{N}) = 180^\circ - m(\hat{M}) - m(\hat{P}) = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$

$$\text{Th. P. } PN^2 = MN^2 + MP^2 = (5\sqrt{3})^2 + 25 = 25 \cdot 3 + 25 =$$

$$= 25(3+1) = 25 \cdot 4 = 100$$

$$PN = \sqrt{100} = 10$$

9.



$\triangle MNP$, $\operatorname{tg}(\hat{N}) = 0,5$

Deci $MQ \perp NP$
 MQ mediană $\Rightarrow \triangle MNP$ - \triangle isoscel \Rightarrow
 $\Rightarrow MN = MP = 4\sqrt{5}$

$$\triangle MQN$$

$$\operatorname{tg}(\hat{N}) = \frac{MQ}{NQ} = 0,5 = \frac{5}{10} = \frac{1}{2} \Rightarrow \frac{MQ}{NQ} = \frac{1}{2} \Rightarrow NQ = 2MQ$$

$$\text{Th. P. } NQ^2 = MN^2 - MQ^2 \Rightarrow NQ^2 = (4\sqrt{5})^2 - MQ^2 \Rightarrow NQ = \sqrt{90 - MQ^2}$$

$$\Rightarrow 2MQ = \sqrt{90 - MQ^2} \Rightarrow 4MQ^2 = 90 - MQ^2 \Rightarrow$$

$$\Rightarrow 5MQ^2 = 90 \Rightarrow MQ^2 = \frac{90}{5} = 18 \Rightarrow MQ = 4$$

$$NQ = 2 \cdot MQ = 2 \cdot 4 = 8$$

$$P_{\triangle MNP} = MN + NP + MP = 4\sqrt{5} + 8 + 8 + 4\sqrt{5} = 16 + 8\sqrt{5} = 8(2 + \sqrt{5})$$