# Olimpiada Națională
# Gazeta Matematică
# Clasa a VII-a

## Model subiect
## Etapa I / Etapa a II-a

1. $a = -12,37 \Rightarrow [a] = [-12,37] = -13$

$\{a\} = \{-12,37\} = -12,37 - (-13) = -12,37 + 13 = 0,63$

R: C. 0,63

2. $n = ?$

$\frac{n+1}{3n+4}$ fracție ireductibilă $\Rightarrow n \in \mathbb{N}$, deoarece

pentru $n = 6 \Rightarrow \frac{6+1}{3\cdot 6+4} = \frac{7}{18+4} = \frac{7}{22}$, $(7,22) = 1$.

3. $\frac{1}{0,0(5)} + \frac{1}{0,0(05)} + \frac{1}{0,0(005)} + \frac{1}{0,0(0005)} = \frac{1}{\frac{5}{90}} + \frac{1}{\frac{5}{990}} + \frac{1}{\frac{5}{9990}} + \frac{1}{\frac{5}{99990}} =$

$= \frac{90}{5} + \frac{990}{5} + \frac{9990}{5} + \frac{99990}{5} = \frac{111\,060}{5} = 22\,212$

R: A. 22212

4. $a = \left(-\frac{3}{5}\right)^8 \cdot \left(-\frac{3}{5}\right)^9 : \left(-\frac{3}{5}\right)^{2^4} \cdot (-1)^{n(n+1)}$, $n \in \mathbb{N}$

$a = \left(-\frac{3}{5}\right)^{8+9-2^4} \cdot (-1)^{n(n+1)}$, $n \in \mathbb{N}$.

$a = \left(-\frac{3}{5}\right)^{17-16} \cdot 1$, $(\forall)\, n \in \mathbb{N}$ (indiferent ce valoare îi dăm lui $n$ avem $(-1)$ la putere pară).

$a = -\frac{3}{5} \cdot 1 \Rightarrow a = -\frac{3}{5}$

$a^{-1} = \left(-\frac{3}{5}\right)^{-1} = -\frac{5}{3}$

R: B. $-\frac{5}{3}$

5. $a = 2^{61}$ ; $b = 3^{41}$ , $n = ?$

$$\frac{2^{3n+1}}{3^{2n+1}} > \frac{a}{b} \Rightarrow \frac{2^{3n+1}}{3^{2n+1}} > \frac{2^{61}}{3^{41}} \Rightarrow \frac{2^{3n}\cdot 2}{3^{2n}\cdot 3} > \frac{2^{61}}{3^{41}} \Big| \cdot \frac{3}{2} \Rightarrow$$

$$\Rightarrow \frac{2^{3n}}{3^{2n}} > \frac{2^{60}}{3^{40}} \Rightarrow \left(\frac{2^3}{3^2}\right)^n > \left(\frac{2^3}{3^2}\right)^{20}. \Rightarrow \left(\frac{8}{9}\right)^n > \left(\frac{8}{9}\right)^{20}.$$

$\frac{8}{9} \rightarrow$ fracție subunitară $\Rightarrow \left(\frac{8}{9}\right)^n > \left(\frac{8}{9}\right)^{20}$ dacă

și numai dacă $n < 20 \Rightarrow n \in \{0, 1, 2, 3, \ldots, 19\}$

R: B. $\{0, 1, 2, \ldots, 19\}$

6. $|\pi - \sqrt{10}| + |\pi + \sqrt{10}| = \sqrt{10} - \cancel{\pi} + \cancel{\pi} + \sqrt{10} = 2\sqrt{10}$

$\left.\begin{array}{l}\pi \simeq 3,14 \\ \sqrt{10} \simeq 3,16\end{array}\right| \Rightarrow \pi < \sqrt{10}.$

R: B. $2\sqrt{10}$.

7. $\sqrt{1\cdot 2\cdot 3\cdot \ldots \cdot 2022 + 2022} \in \mathbb{R}\setminus\mathbb{Q}$

$1\cdot 2\cdot 3\cdot \ldots \cdot 2022 = 2022!$

$u(2022!) = 0$, deoarece avem produs de $2\cdot 5$, $10, 20, \ldots$

$u(2022! + 2022) = u(2022!) + u(2022) = 0 + 2 = 2$.

Știm că numărul care are ultima cifră 2 nu este pătrat perfect, deci radical din numărul nostru aparține mulțimii $\mathbb{R}\setminus\mathbb{Q}$, este număr irațional.

R: A. $\mathbb{R}\setminus\mathbb{Q}$

8. $n = \overline{200xyz5} = a^2$

$\sqrt{n} = ?$

Presupunem că $n = 2000005 \Rightarrow \sqrt{n} = \sqrt{2000005} \simeq 1414 \Rightarrow$

$\Rightarrow \sqrt{n} = 1415$, pentru că $n$ se termină în cifra 5.

$1+4+1+5 = 11$

R: B. 11

9. $a = \sqrt{4+\sqrt{7}} - \sqrt{4-\sqrt{7}}$

$$\sqrt{x \pm \sqrt{y}} = \sqrt{\frac{x+c}{2}} \pm \sqrt{\frac{x-c}{2}}, \quad c = \sqrt{x^2 - y}$$

$$\sqrt{4+\sqrt{7}} = \sqrt{\frac{4+3}{2}} + \sqrt{\frac{4-3}{2}} = \sqrt{\frac{7}{2}} + \sqrt{\frac{1}{2}} = \frac{\sqrt{7}}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{\sqrt{7}+1}{\sqrt{2}}$$

$$c = \sqrt{4^2 - 7} = \sqrt{16-7} = \sqrt{9} = 3$$

$$\sqrt{4-\sqrt{7}} = \sqrt{\frac{4+3}{2}} - \sqrt{\frac{4-3}{2}} = \sqrt{\frac{7}{2}} - \sqrt{\frac{1}{2}} = \frac{\sqrt{7}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{\sqrt{7}-1}{\sqrt{2}}$$

$$a = \frac{\sqrt{7}+1}{\sqrt{2}} - \frac{\sqrt{7}-1}{\sqrt{2}} = \frac{\cancel{\sqrt{7}}+1-\cancel{\sqrt{7}}+1}{\sqrt{2}}$$

$$a = \frac{^{\sqrt{2})}2}{\sqrt{2}} \Rightarrow a = \frac{\cancel{2}^{1}\sqrt{2}}{\cancel{2}} \Rightarrow a = \sqrt{2}$$

sau calculăm $a^2$ și apoi calculăm pe $a$. R: C. $\sqrt{2}$

10. $n = ?$, $n \in \mathbb{N}^*$

$\frac{2\sqrt{n} - 5\sqrt{3}}{\sqrt{3}+\sqrt{n}} \in \mathbb{Z} \Rightarrow (2\sqrt{n} - 5\sqrt{3}) \vdots (\sqrt{3} + \sqrt{n})$

Pentru $n = 15 \Rightarrow (2\sqrt{15} - 5\sqrt{3}) \vdots (\sqrt{3} + \sqrt{15})$ (F)

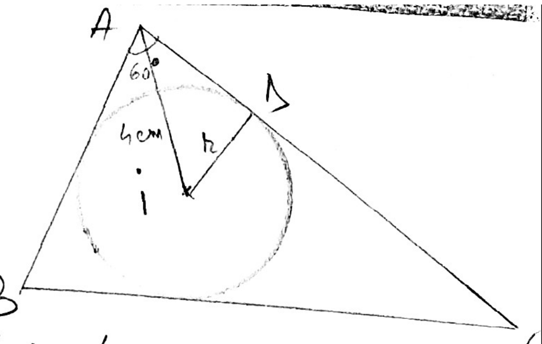
$n = 121 \Rightarrow (2\sqrt{121} - 5\sqrt{3}) \vdots (\sqrt{3} + \sqrt{121}) \Rightarrow (22 - 5\sqrt{3}) \vdots (\sqrt{3} + 11)$ (F)

$n = 36 \Rightarrow (2\sqrt{36} - 5\sqrt{3}) \vdots (\sqrt{3} + \sqrt{36}) \Rightarrow (12 - 5\sqrt{3}) \vdots (\sqrt{3} + 6)$ (F)

$n = 108 \Rightarrow (2\sqrt{108} - 5\sqrt{3}) \vdots (\sqrt{3} + \sqrt{108}) \Rightarrow (12\sqrt{3} - 5\sqrt{3}) \vdots (\sqrt{3} + 6\sqrt{3}) \Rightarrow$

$\Rightarrow 7\sqrt{3} \vdots 7\sqrt{3}$ (A) $\Rightarrow n = 108$ R: D. 108.

11. $\triangle ABC$, $\sphericalangle BAC = 60^\circ$




$AI = 4$ cm

$d(I, BC) = ID = r$

$I$ centrul cercului înscris $\triangle ABC \Rightarrow AI$ - bisectoarea $\sphericalangle BAC \Rightarrow$

$\Rightarrow \sphericalangle IAD = \sphericalangle BAC : 2 \Rightarrow \sphericalangle IAD = 60^\circ : 2 \Rightarrow \sphericalangle IAD = 30^\circ$.

$ID \perp BC \Rightarrow \triangle AID$ - $\triangle$ dreptunghic | $30^\circ - 60^\circ - 90^\circ$ $\Rightarrow ID = AI : 2 \Rightarrow$

$\sphericalangle IAD = 30^\circ$

$\Rightarrow ID = 4\text{ cm} : 2 \Rightarrow ID = 2\text{ cm} = r$.

$d(I, BC) = 2$ cm

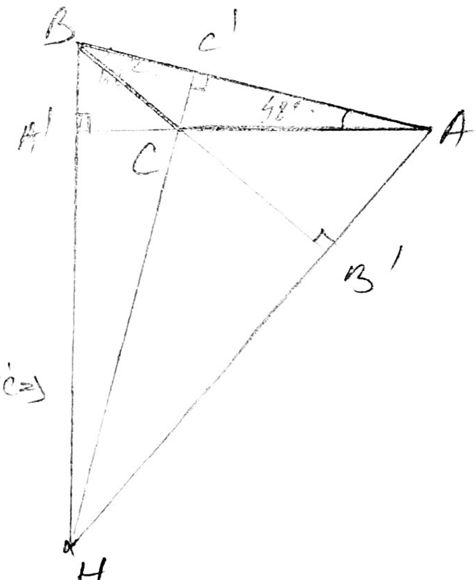
R: C. 2cm.

12. $\triangle ABC$ - obtuzunghic

$\sphericalangle BAC = 48^\circ$

$H$ ortocentrul

$\sphericalangle BHC = ?$




$H$ ortocentrul $\Rightarrow H$ este intersecția celor 3 înălțimi ale $\triangle ABC$.

$AA' \perp BH \Rightarrow \triangle AA'B$ - dreptunghic $\Rightarrow$

$\Rightarrow \sphericalangle A'AB + \sphericalangle ABA' = 90^\circ$.

$\sphericalangle ABA' = 90^\circ - 48^\circ = 42^\circ$.

$HC' \perp AB \Rightarrow \triangle HC'B$ - dreptunghic $\Rightarrow$

$\Rightarrow \sphericalangle C'BH + \sphericalangle C'HB = 90^\circ \Rightarrow \sphericalangle C'HB = 90^\circ - 42^\circ = 48^\circ \Rightarrow$

$\Rightarrow \sphericalangle BHC = 48^\circ$

R: A. 48°.

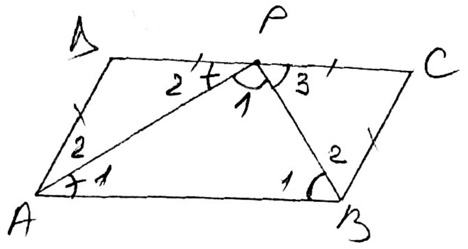
13. $ABCD$ - paralelogram

$AB = 2BC$

$P$ mijlocul $[CD] \Rightarrow PC \equiv PD$

---

$\sphericalangle APB = ?$




---

$ABCD$ paralelogram $\Rightarrow AB \equiv DC \Rightarrow AB = DC = 2BC$

$P$ mijlocul $[DC] \Rightarrow PC \equiv PD \Rightarrow PC = PD = DC : 2$ $\Bigg| \Rightarrow$

$\Rightarrow PC = BC \Rightarrow \triangle PCB$ - isoscel $\Rightarrow \sphericalangle P_3 \equiv \sphericalangle B_2$

$DP = AD \Rightarrow \triangle ADP$ - isoscel $\Rightarrow \sphericalangle P_2 \equiv \sphericalangle A_2$

$DC \parallel AB$, $PB$ - secantă $\Big| \Rightarrow \sphericalangle P_3 \equiv \sphericalangle B_1$ ($\sphericalangle$ alterne interne)

$DC \parallel AB$, $AP$ - secantă $\Big| \Rightarrow \sphericalangle P_2 \equiv \sphericalangle A_1$ ($\sphericalangle$ alterne interne

În $\triangle APB$

$\sphericalangle APB = 180° - (\sphericalangle A_1 + \sphericalangle B_1)$

În $\triangle ADP$ - isoscel

$\sphericalangle P_2 = \dfrac{180° - \sphericalangle D}{2}$

În $\triangle PCB$ - isoscel $\Rightarrow \sphericalangle P_3 = \dfrac{180° - \sphericalangle C}{2}$ $\Bigg| \Rightarrow$

$\Rightarrow \sphericalangle P_2 + \sphericalangle P_3 = \dfrac{180° - \sphericalangle D}{2} + \dfrac{180° - \sphericalangle C}{2} = \dfrac{360° - (\sphericalangle D + \sphericalangle C)}{2}$

$\sphericalangle D + \sphericalangle C = 180°$ $\Bigg| \Rightarrow$

$\Rightarrow \sphericalangle P_2 + \sphericalangle P_3 = \dfrac{360° - 180°}{2} = \dfrac{180°}{2} = 90°$.

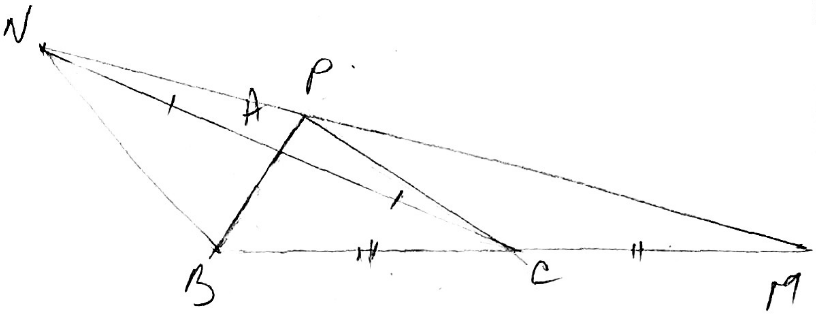
dar. $\sphericalangle P_2 \equiv \sphericalangle A_1$ și $\sphericalangle$ $\sphericalangle P_3 = \sphericalangle B_1$ $\Bigg| \Rightarrow$

$\Rightarrow \sphericalangle A_1 + \sphericalangle B_1 = 90°$.

Atunci, $\sphericalangle APB = 180° - 90° = 90°$.

R: D. $90°$.

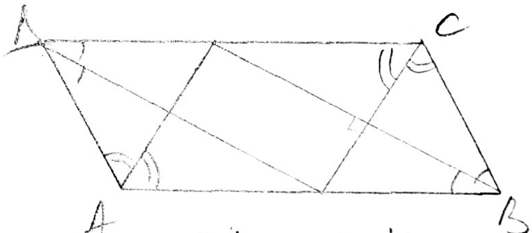
14. $\triangle ABC$ - oarecare
$[CM] \equiv [BC]$
$[AN] \equiv [CA]$
$AB \cap MN = \{P\}$




Dacă desenul este construit corect, din măsurători reiese că: $AB = 3AP$.

R: B. $AB = 3AP$.

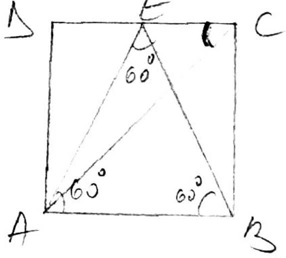
15. $ABCD$ - paralelogram
$AB > AD$




Din realizarea desenului se observă că intersecția bisectoarelor determină un dreptunghi.

R: C. dreptunghi

16. $ABCD$ - pătrat
$\triangle ABE$ - echilateral
$\sphericalangle ACE = ?$




---

$ABCD$ - pătrat
$AC$ diagonala pătratului $\Big| \Rightarrow AC$ - bisectoarea $\sphericalangle A$ și $\sphericalangle C \Rightarrow$
$\Rightarrow \sphericalangle ACE = \sphericalangle C : 2 \Rightarrow \sphericalangle ACE = 90° : 2 = 45°$

R: C. 45°.

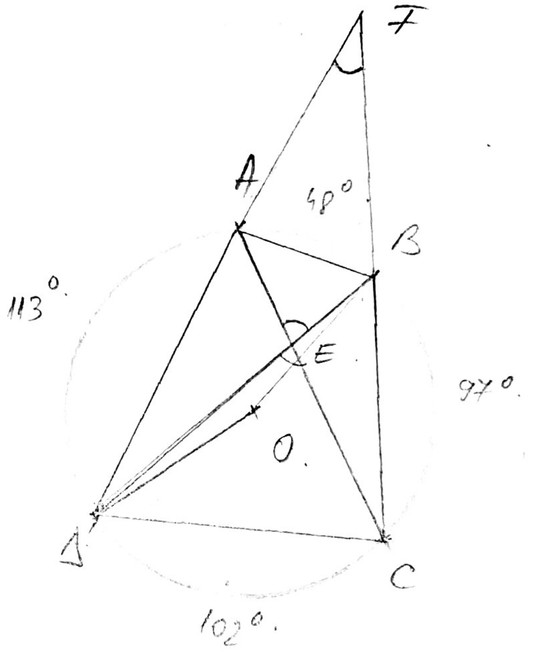
17. trapezul isoscel cu diagonalele perpendiculare are înălțimea egală cu linia mijlocie.

$A = h \cdot l_m = a \cdot a = a^2$

$h$ = înălțimea trapezului
$l_m$ = linia mijlocie a trapezului

R: D. $a^2$

18. ABCD - patrulater.
$\overset{\frown}{AB} = 48^\circ$
$\overset{\frown}{BC} = 97^\circ$
$\overset{\frown}{DC} = 102^\circ$.




$\sphericalangle AEB = \dfrac{\overset{\frown}{AB} + \overset{\frown}{DC}}{2} = \dfrac{48^\circ + 102^\circ}{2} = \dfrac{150^\circ}{2} = 75^\circ$.

R: C. $75^\circ$.

19. $\sphericalangle(AD, BC) = \sphericalangle AFB$.

$\sphericalangle AFB = \dfrac{\overset{\frown}{DC} - \overset{\frown}{AB}}{2} = \dfrac{102^\circ - 48^\circ}{2} = \dfrac{54^\circ}{2} = 27^\circ$.

20. $\sphericalangle BDO = ?$

$\Delta BDO$ - isoscel, deoarece $OD = OB = R \Rightarrow \sphericalangle BDO \equiv \sphericalangle DBO$

$\sphericalangle DOB = \overset{\frown}{DAB} = 113^\circ + 48^\circ = 161^\circ$.

$\overset{\frown}{AD} = 360^\circ - (\overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{DC}) = 360^\circ - 247^\circ = 113^\circ$.

$\sphericalangle BDO + \sphericalangle DBO + \sphericalangle DOB = 180^\circ \Rightarrow 2\sphericalangle BDO = 180^\circ - 161^\circ$

$\sphericalangle BDO = 19^\circ : 2 = 18^\circ 60' : 2 = 9^\circ 30'$

R: D. $9^\circ 30'$

21. $\triangle ABC$ - dreptunghic

$\sphericalangle B = 90^\circ$

$\sphericalangle C \neq 30^\circ$

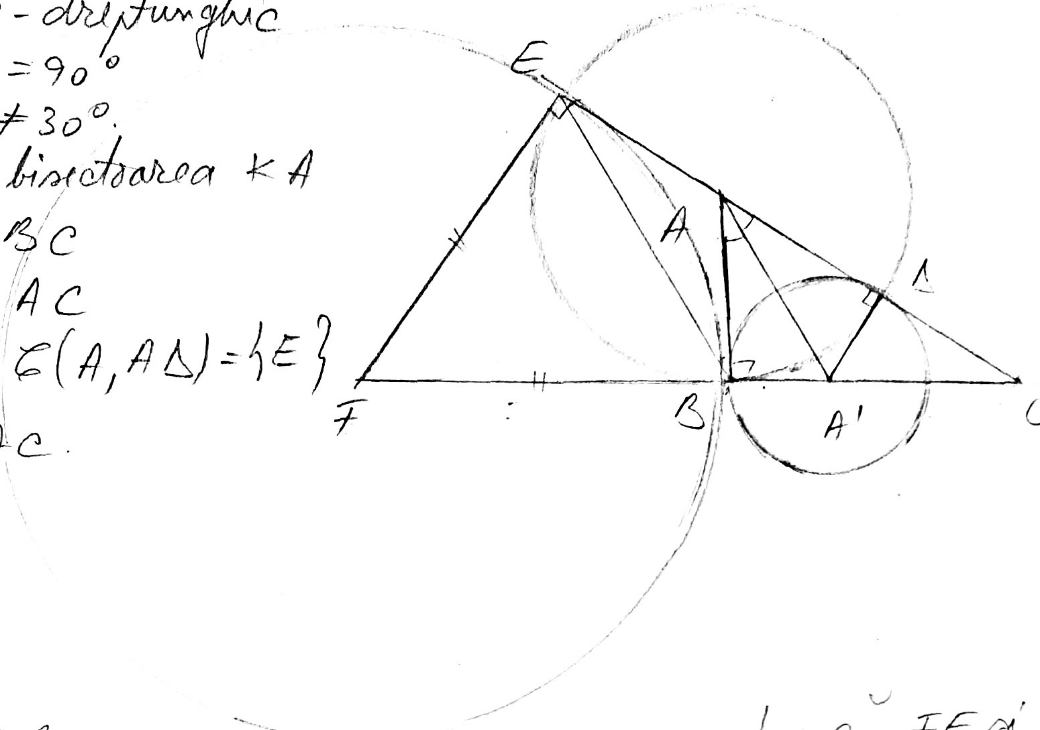
$AA'$ - bisectoarea $\sphericalangle A$

$A' \in BC$

$A'D \perp AC$

$AC \cap \mathcal{C}(A, AD) = \{E\}$

$FE \perp AC$.




$\triangle EFB$ - isoscel în vârful $F$, pentru că $FE$ și $FB$ sunt tangente la cercul $\mathcal{C}(A, EA)$, deci $FE \equiv FB$.

R: B. isoscel de vârf F.

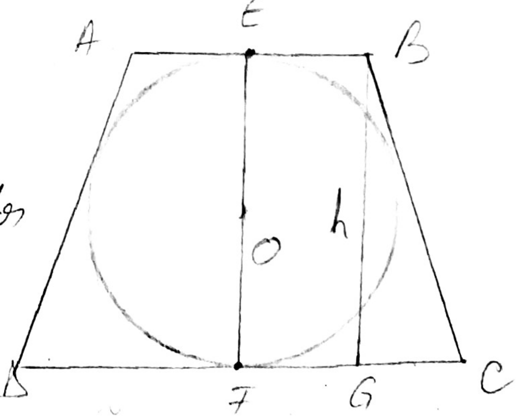
22. $\mathcal{C}(A', A'D)$ și $\mathcal{C}(F, FE) \Rightarrow$ tangente exterior, deoarece

$\mathcal{C}(A', A'D) \cap \mathcal{C}(F, FE) = \{B\}$, unde $FE = FB$, $A'D = A'B$

și $d(F, A') = FA' = FB + BA'$

R: C. tangente exterior

23. Într-un trapez isoscel se poate înscrie un cerc dacă și numai dacă suma bazelor este egală cu suma lungimilor laturilor neparalele.




$$\left.\begin{array}{l} AB + DC = AD + BC \\ AD = BC \end{array}\right| \Rightarrow 2BC = AB + DC$$

$$BC = \frac{AB + DC}{2}$$

R: C. $\frac{AB + CD}{2}$

24. Diametrul cercului $\mathcal{C}(O, r)$ este chiar înălțimea trapezului isoscel $\Rightarrow d = h \Rightarrow EF = BG$.

$BG \perp DC \Rightarrow \triangle BGC$ - dreptunghic $\xrightarrow{T. Pitagora} BG^2 + GC^2 = BC^2$

$GC = (DC - AB) : 2$

$BC = \frac{AB + DC}{2}$.

$\Rightarrow$

$$\Rightarrow BG^2 = \left(\frac{AB+DC}{2}\right)^2 - \left(\frac{DC-AB}{2}\right)^2$$

$$BG^2 = \left(\frac{AB+DC}{2} - \frac{DC-AB}{2}\right)\left(\frac{AB+DC}{2} + \frac{DC-AB}{2}\right)$$

Am folosit formula $a^2 - b^2 = (a-b)(a+b)$

$$BG^2 = \frac{AB + \cancel{DC} - \cancel{DC} + AB}{2} \cdot \frac{\cancel{AB} + DC + DC - \cancel{AB}}{2}$$

$$BG^2 = \frac{\cancel{2}AB}{\cancel{2}} \cdot \frac{\cancel{2}DC}{\cancel{2}} \Rightarrow BG^2 = AB \cdot DC$$

$$BG = \sqrt{AB \cdot DC} = d$$

R: D. $\sqrt{AB \cdot CD}$