Varianta 1.

Sub. I

1. $C_3^2 + 3! = \frac{3!}{2!(3-2)!} + 3! = \frac{3}{1!} + 3! = 3 + 6 = 9$

2. $\log_5(3x+4) = 2 \Leftrightarrow \log_5(3x+4) = \log_5 5^2 \Leftrightarrow$

$\Leftrightarrow 3x+4 = 25 \Leftrightarrow 3x = 21 \Leftrightarrow x = 7 \in$

c.e: $3x+4>0 \Rightarrow x > -\frac{4}{3}$

3. $\frac{1}{x_1} + \frac{1}{x_2} = ?$ $\qquad x^2 - x - 2 = 0$

$\Delta = b^2 - 4ac = 1 + 8 = 9$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 3}{2} \Rightarrow x_1 = 2, \quad x_2 = -1.$

$\frac{1}{2} - \frac{\overset{2}{1}}{1} = \frac{1-2}{2} = -\frac{1}{2}$

sau

$\begin{cases} x_1 + x_2 = -\frac{b}{a} = 1 \\ x_1 \cdot x_2 = \frac{c}{a} = -2 \end{cases}$ Rel. lui Viete

$\frac{\overset{x_2)}{1}}{x_1} + \frac{\overset{x_1)}{1}}{x_2} = \frac{x_2 + x_1}{x_1 \cdot x_2} = \frac{1}{-2} = -\frac{1}{2}$

4. $\left.\begin{array}{l} f(0) = 0 \\ f(1) = -1 \end{array}\right| \Rightarrow Im\, f = [-1, 0]$

5. $\vec{AB} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} =$ $\qquad \vec{A} = x_1\vec{i} + y_1\vec{j}$

$= -3\vec{i} + 4\vec{j}$ $\qquad \vec{B} = x_2\vec{i} + y_2\vec{j}.$

$\left.\vec{AB} = a\vec{i} + b\vec{j}\right| \Rightarrow \begin{array}{l} a = -3 \\ b = 4 \end{array}$

6. Din th. lui Pitagora generalizată avem:

$\cos\hat{B} = \frac{AB^2 + BC^2 - AC^2}{2AB \cdot BC} = \frac{16+3-7}{8\sqrt{3}} = \frac{\cancel{12}^{3}}{\cancel{8}_{2}\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{6}_2} = \frac{\sqrt{3}}{2} \Rightarrow \cos\hat{B} = \frac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow m(\measuredangle B) = 30°.$

Subiectul II

1. $d = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \end{vmatrix}$, x_1, x_2, x_3 sol ec $x^3 - 3x + 2 = 0$.

a) $x_1 + x_2 + x_3 = ?$

Rel. lui Vieté $\quad x_1 + x_2 + x_3 = -\frac{b}{a} = \frac{0}{1} = 0$

b) $x_1^3 + x_2^3 + x_3^3 = -6$?

$x^3 - 3x + 2 = 0 \Rightarrow \begin{cases} x_1^3 - 3x_1 + 2 = 0 \\ x_2^3 - 3x_2 + 2 = 0 \\ x_3^3 - 3x_3 + 2 = 0 \end{cases}$

$x_1^3 + x_2^3 + x_3^3 - 3\underbrace{(x_1 + x_2 + x_3)}_{0} + 6 = 0 \Rightarrow x_1^3 + x_2^3 + x_3^3 = -6$

c) $d = ?$

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \\ x_3 & x_1 & x_2 \\ x_1 & x_2 & x_3 \\ x_2 & x_3 & x_1 \end{vmatrix} = x_1 \cdot x_2 x_3 + x_1 x_2 x_3 + x_1 x_2 x_3 - x_3^3 - x_1^3 - x_2^3 =$$

$$= 3x_1 x_2 x_3 - (x_1^3 + x_2^3 + x_3^3) = 3 \cdot (-2) - (-6) =$$

$$= -6 + 6 = 0.$$

Rel Viete: $x_1 x_2 x_3 = -\frac{d}{a} = -2$

2. $x \circ y = xy + 4x + 4y + 12$

a) $x \circ y = (x+4)(y+4) - 4$?

$(x+4)(y+4) - 4 = xy + 4x + 4y - 4 + 16 = xy + 4y + 4x + 12$.

$\Rightarrow x \circ y = xy + 4x + 4y + 12$

b) $x \circ (-4) = x(-4) + 4x + 4(-4) + 12 = \cancel{-4x} \cancel{+4x} - 16 + 12 = -4, \forall x \in R$

c) $(-2009) \circ (-2008) \circ \underset{(-4)}{\ldots} \circ 2008 \circ 2009 = ?$ Observăm că $(-4) \circ x = -4, \forall x \in R$

$\Rightarrow (-2009) \circ (-2008) \circ \ldots \circ (-4) \circ \ldots \circ 2008 \circ 2009 = -4$

Subiectul III

1. $f:\mathbb{R}\setminus\{-1\}\to\mathbb{R}$, $f(x)=\dfrac{x^2}{x+1}$

a) $f'(x)=?$

$$f'(x)=\frac{2x(x+1)-x^2}{(x+1)^2}=\frac{2x^2-x^2+2x}{(x+1)^2}=\frac{x^2+2x}{(x+1)^2}=\frac{x(x+2)}{(x+1)^2}.$$

b) De. $f'(x)=\dfrac{x(x+2)}{(x+1)^2}$.

$x(x+2)=0 \Rightarrow x_1=0$
$x_2=-2$.

x	$-\infty$		-2		-1		0		$+\infty$
$f'(x)$		$+$	0	$-$	|	$-$	0	$+$	
$f(x)$		$\nearrow$	-4	$\searrow$	|	$\searrow$	0	$\nearrow$	

$f(0)=0.$
$f(-2)=-4$

Pe intervalele $(-\infty;-2]$ și $[0,+\infty)$ fct. este crescătoare

Pe intervalele $[-2,-1)$ și $(-1,0]$ fct. este descrescătoare.

c) $f(-2)=\dfrac{4}{-1}=-4$.

din tabel obs. că $f(x)\le -4$ pt $\forall\ x<-1$.

2. $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\begin{cases} x^2+e^x, & x\le 0\\ \sqrt{x}+1, & x>0\end{cases}$

a) Studiem continuitatea fct. f în pct $x=0$.

$l_s=\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}(x^2+e^x)=e^0=1.$

$l_d=\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}(\sqrt{x}+1)=1.$

$f(0)=e^0=1$

$\Rightarrow f(x)$ este continuă în $x=0$ deci este continuă pe $\mathbb{R}$ $\Rightarrow$ că funcția are primitive pe $\mathbb{R}$

b) $\int_{-1}^{0} x f(x)\,dx = ?$

=4= Var: I

$$\int_{-1}^{0} x f(x)\,dx = \int_{-1}^{0} x(x^2+e^x)\,dx = \int_{-1}^{0}(x^3+xe^x)\,dx = \int_{-1}^{0} x^3\,dx +$$

$$+\int_{-1}^{0} xe^x\,dx = \frac{x^4}{4}\Big|_{-1}^{0} + xe^x\Big|_{-1}^{0} - \int_{-1}^{0} e^x\,dx = -\frac{1}{4}+e^{-1}-e^x\Big|_{-1}^{0} =$$

$f(x)=x \quad f'(x)=1$
$g(x)=e^x \quad g(x)=e^x.$

$$= -\frac{1}{4}+e^{-1}-1+e^{-1} = -\frac{1}{4}-\frac{4}{4}2e^{-1} =$$

$$= -\frac{5}{4}+2e^{-1} = \frac{8e^{-1}-5}{4} = \frac{8-5e}{4e}.$$

c) $V=?$ $\quad g:[0;1]\to\mathbb{R}, \; g(x)=f(x)$

$$V(C_g) = \pi\int_0^1 f^2(x)\,dx = \pi\int_0^1 g^2(x)\,dx = \pi\int_0^1(\sqrt{x}+1)^2\,dx =$$

$$= \pi\int_0^1(x+2\sqrt{x}+1)\,dx = \pi\left[\int_0^1 x\,dx + 2\int_0^1\sqrt{x}\,dx + \int_0^1 dx\right] =$$

$$= \pi\left(\frac{x^2}{2}\Big|_0^1 + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^1 + x\Big|_0^1\right) = \pi\left(\frac{1}{2}+2\cdot\frac{2}{3}+\frac{2}{1}\right) = \pi\left(\frac{3}{2}+\frac{4}{3}\right) = \frac{17}{6}\pi$$