Variantа 2

Subiectul I.

1. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x - 3$, $f(-4) \cdot f(-3) \cdot \ldots \cdot f(3) \cdot f(4) = ?$

$f(-4) = -4 - 3 = -7$ $\quad$ $f(1) = 1 - 3 = -2$

$f(-3) = -3 - 3 = -6$ $\quad$ $f(2) = 2 - 3 = -1$

$f(-2) = -2 - 3 = -5$ $\quad$ $f(3) = 3 - 3 = 0.$

$f(-1) = -1 - 3 = -4$ $\quad$ $f(4) = 4 - 3 = 1.$

$f(0) = -3$

$(-7) \cdot (-6) \cdot (-5) \cdot (-4) \cdot (-3) \cdot (-2) \cdot (-1) \cdot 0 \cdot 1 = 0.$

2. $\log_2 (x+2) + \log_2 x = 3 \Leftrightarrow \log_2 [x(x+2)] = \log_2 2^3 \Leftrightarrow$

$\Leftrightarrow x(x+2) = 2^3 \Leftrightarrow x^2 + 2x - 8 = 0.$ $\quad$ C.E. $\begin{cases} x+2>0 \\ x>0. \end{cases} \Rightarrow$

$\Delta = 4 + 32 = 36$

$x_{1,2} = \dfrac{-2 \pm 6}{2}$ $\to x_1 = 2$, $\to x_2 = -4 \notin (0, \infty)$ $\quad \Rightarrow x > 0$

$\to$ sol. finală $x = 2$.

3. $x^2 - 5x + 5 \leq 1 \Leftrightarrow x^2 - 5x + 4 \leq 0.$

$x^2 - 5x + 4 = 0$

$\Delta = 25 - 16 = 9 \Rightarrow x_{1,2} = \dfrac{5 \mp 3}{2}$ $\to x_1 = 1$, $\to x_2 = 4$

x	$-\infty$		1		4		$+\infty$
x^2-5x+4	+	+	0	− −	0	+	+

$x \in [1, 4] \Rightarrow x \in \{1, 2, 3, 4\}$

4. $\forall x \in \mathbb{R}$. $\underbrace{3^x - 1}_{=a_1}, \underbrace{3^{x+1}}_{=a_2}, \underbrace{5 \cdot 3^x + 1}_{=a_3}$. sunt termeni consecutivi în $\div$

$a_n = \dfrac{a_{n-1} + a_{n+1}}{2} \Rightarrow a_2 = \dfrac{a_1 + a_3}{2} \Rightarrow a_2 = \dfrac{3^x - \cancel{1} + 5 \cdot 3^x + \cancel{1}}{2} =$

$= \dfrac{3^x + 5 \cdot 3^x}{2} = \dfrac{3^x(1+5)}{2} = \dfrac{3^x \cdot \cancel{6}^{3}}{\cancel{2}} = 3^x \cdot 3 = 3^{x+1}$

=6 = −6 =

5. $A(4,-8)$, $B(6,3)$

$\vec{OA} + \vec{OB} = \underbrace{4\vec{i} - 8\vec{j}}_{\vec{OA}} + \underbrace{6\vec{i} + 3\vec{j}}_{\vec{OB}} = 10\vec{i} - 5\vec{j}$

deci vectorul $\vec{OA} + \vec{OB}\,(10, -5)$

6. $\triangle ABC$.

$AC = 2$.

$m(\sphericalangle BAC) = 30^\circ$.

$AB = 4$,

$S_{\triangle ABC} = ?$

Aplicăm formula: $\boxed{S = \frac{b \cdot c \sin \hat{A}}{2}}$

$S = \frac{AC \cdot AB \cdot \sin A}{2} = \frac{\cancel{2} \cdot 4 \cdot \sin 30^\circ}{\cancel{2}} = \cancel{4}^{2} \cdot \frac{1}{\cancel{2}} = 2$

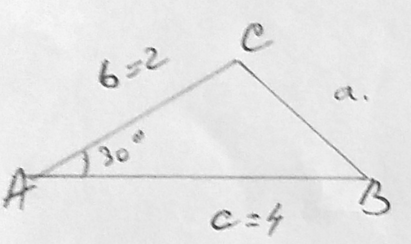
$S = 2$

Dacă cunoșteam 1) $\sphericalangle B$ și laturile $BC, AB \Rightarrow$

$\Rightarrow \boxed{S = \frac{a \cdot c \sin \hat{B}}{2}}$

2) $\sphericalangle C$ și laturile $AC, BC \Rightarrow$

$\Rightarrow \boxed{S = \frac{a \cdot b \cdot \sin \hat{C}}{2}}$



Aceste formule se aplică în orice triunghi.

Subiectul II

1. $d = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, $a, b \in \mathbb{R}$

a) $d = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 8 - \cancel{1} + \cancel{1} + 2 + 2 + 2 = 14$

b) $d = \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = \frac{1}{2}(a+b+c)(a^2 - 2ab + b^2 + b^2 - 2bc +$

$+ c^2 + c^2 - 2ac + a^2) = \frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac) =$

$= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac) = a^3 + \cancel{ab^2} + \cancel{ac^2} - \cancel{a^2b} - abc - \cancel{a^2c} +$

$+ \cancel{a^2b} + b^3 + \cancel{bc^2} - \cancel{ab^2} - \cancel{b^2c} - abc + \cancel{a^2c} + \cancel{b^2c} + c^3 - abc - \cancel{bc^2} - \cancel{ac^2} =$

$= a^3 + b^3 + c^3 - 3abc$.

$$d = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = a^3+b^3+c^3-abc-abc-abc = a^3+b^3+c^3-3abc$$

c) $$\begin{vmatrix} 2^x & 3^x & 5^x \\ 5^x & 2^x & 3^x \\ 3^x & 5^x & 2^x \end{vmatrix} = 0 \Leftrightarrow 2^{3x}+3^{3x}+5^{3x}-3(2^x\cdot 3^x\cdot 5^x)=0 \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2}(2^x+3^x+5^x)\left[(2^x-3^x)^2+(3^x-5^x)^2+(5^x-2^x)^2\right]=0.$$

$$2^x+3^x+5^x=0.$$

Cum $2^x+3^x+5^x>0 \Rightarrow 2^x=3^x=5^x \Rightarrow x=0.$

2. $x\circ y = 2xy-6x-6y+21.$

a) $x\circ y = 2(x-3)(y-3)+3 = , \ \forall x,y\in \mathbb{R}.$

$= 2(xy-3x-3y+9)+3 = 2xy-6x-6y+18+3 = 2xy-6x-6y+21.$

b) $x\circ x = 11 \Leftrightarrow 2x^2-12x+21=11 \Leftrightarrow 2x^2-12x+10=0 \ /:2 \Leftrightarrow$

$\Leftrightarrow x^2-6x+5=0 \Leftrightarrow (x-5)(x-1)=0 \Rightarrow x_1=5$

$x_2=1$

c) $1\circ\sqrt{2}\circ\sqrt{3}\circ \ldots \circ\sqrt{2009} = ?$; $\sqrt{9}=3$ iar x = orice altă valoare

$x\circ 3 = 3\circ x = 6x-6x-18+21 = 3, \ \forall x\in\mathbb{R} \Rightarrow 1\circ\sqrt{2}\circ\sqrt{3} \ldots \circ\sqrt{2009} =$

Subiectul III

1. $f:\mathbb{R}\to\mathbb{R}, \ f(x)=e^x-e^{-x}.$

a) $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x} = f'(0)$

$f'(x)=e^x+e^{-x}$

$f'(0)=1+1=2.$

b) $f'(x) = e^x + e^{-x} > 0, \forall x \in \mathbb{R} \Rightarrow f$ este crescătoare pe $\mathbb{R}$.

c) $S = g(0) + g(1) + \dots + g(2009)$, $g: \mathbb{R} \to \mathbb{R}$, $g(x) = f'(x) - f''(x)$

$f''(x) = e^x - e^{-x}$

$g(x) = \cancel{e^x} + e^{-x} - \cancel{e^x} + e^{-x} = 2e^{-x}$.

$S = 2e^{-0} + 2e^{-1} + \dots + 2e^{-2009} = 2\underbrace{\left(1 + \frac{1}{e} + \frac{1}{e^2} + \dots + \frac{1}{e^{2009}}\right)}_{S_1} \Rightarrow$

avem $n = 2010$ termeni

$q = \frac{1}{e} \Rightarrow S_1$ sunt în $\div\div \Rightarrow S_1 = \frac{b_1(q^n - 1)}{q - 1}$.

$\Rightarrow S = 2 \cdot S_1 = 2 \cdot \frac{(q^{2010} - 1)}{q - 1} = 2 \cdot \frac{\left(\frac{1}{e}\right)^{2010} - 1}{\frac{1}{e} - 1} = 2 \cdot \frac{(1 - e^{2010})e}{e^{2010}(1 - e)}$

$= 2 \cdot \frac{(1 - e^{2010})}{e^{2009}(1 - e)}$

2. $f, F: \mathbb{R} \to \mathbb{R}$, $f(x) = xe^x$, $F(x) = (x-1)e^x$.

a)

F e primitivă a lui f dc. $F' = f$.

$F'(x) = e^x + (x-1)e^x = e^x(\cancel{1} + x \cancel{-1}) = xe^x$

b) $S = ?$
$x = 0$ și $x = 1$ $\Rightarrow S = \int_0^1 f(x)\,dx = F(x)\Big|_0^1 = (x-1)e^x\Big|_0^1 =$

$= 0 - (-1)e^0 = e^0 = 1$.

c) $\int_1^x \frac{f(t)f''(t) - (f'(t))^2}{f^2(t)}\,dt = \frac{x+1}{x} - 2$, $\forall x > 1$. $f(t) = te^t$, $f'(t) = e^t + te^t$, $f''(t) = e^t + e^t + te^t = e^t(2+t)$

$\int_1^x \frac{te^t \cdot e^t(2+t) - (e^t + te^t)^2}{(te^t)^2}\,dt = \int_1^x \frac{\cancel{e^{2t}}(2t + t^2) - \cancel{e^{2t}}(1+t)^2}{\cancel{e^{2t}} \cdot t^2}\,dt =$

$= \int_1^x \frac{\cancel{2t} + \cancel{t^2} - 1 - \cancel{2t} - \cancel{t^2}}{t^2}\,dt = \int_1^x -\frac{1}{t^2}\,dt = -\int_1^x \frac{1}{t^2}\,dt = \frac{1}{t}\Big|_1^x = \frac{1}{x} - 1 =$

$= \frac{1-x}{x}$ $\quad \left(\frac{x+1}{x} - 2 = \frac{x+1-2x}{x} = \frac{1-x}{x}\right)$ c.c.t.d.