Model
2016 – M_tehnologic

Subiectul I

1. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=\frac{1}{4}$?

$\frac{2}{1}-\frac{1}{2}=\frac{2-1}{2}=\frac{1}{2}$.

$\frac{3}{1}-\frac{1}{3}=\frac{3-1}{3}=\frac{2}{3}$

$\frac{4}{1}-\frac{1}{4}=\frac{4-1}{4}=\frac{3}{4}$.

$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}=\frac{1}{4}$

2. $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^2-3x+2$

$G_f\cap Ox \Rightarrow y=0 \Rightarrow f(x)=0$.

$x^2-3x+2=0 \Leftrightarrow x^2-x-2x+2=0 \Leftrightarrow x(x-1)-2(x-1)=0$

$\Leftrightarrow (x-1)(x-2)=0 \Rightarrow \begin{cases} x-1=0 \Rightarrow x_1=1 \\ x-2=0 \Rightarrow x_2=2 \end{cases}$

3. $\log_5(2x-1)=2 \Rightarrow \log_5(2x-1)=\log_5 5^2 \Rightarrow$

c.e. $2x-1>0 \Rightarrow 2x>1 \Rightarrow x>\frac{1}{2}$

$\Rightarrow 2x-1=25 \Rightarrow 2x=25+1 \Rightarrow 2x=26 \Rightarrow x=26:2=13$.

4. $A=\{10,20,30,40,50,60,70,80,90\}$

$P=\frac{\text{nr. cazuri favorabile}}{\text{nr. cazurilor posibile}}=\frac{4}{9}$

divizori ai lui 1000 sunt: 10; 20; 40; 50.

cazuri posibile = 9 (deoarece avem 9 numere)

cazuri favorabile = 4 (deoarece avem 4 numere care îl divid)

5. $O(0,0)$, $A(0,3)$, $B(4,0)$

$P_{\triangle AOB} = ?$

$P_{\triangle AOB} = AO + OB + AB$.

$AO = \sqrt{(x_O - x_A)^2 + (y_O - y_A)^2} = \sqrt{(0-0)^2 + (0-3)^2} = \sqrt{(-3)^2} = \sqrt{9} =$

$= 3$.

$OB = \sqrt{(x_B - x_O)^2 + (y_B - y_O)^2} = \sqrt{(4-0)^2 + (0-0)^2} = \sqrt{4^2} = 4$

$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(4-0)^2 + (0-3)^2} = \sqrt{4^2 + (-3)^2} =$

$= \sqrt{16+9} = \sqrt{25} = 5$

$P_{\triangle AOB} = 3 + 4 + 5 = 12$

6. $\sin x = \frac{3}{5}$?, $x \in \left(0, \frac{\pi}{2}\right)$ și $\cos x = \frac{4}{5}$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 \Rightarrow$

$\Rightarrow \sin^2 x = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{25-16}{25} = \frac{9}{25} \Rightarrow$

$\Rightarrow \sin^2 x = \frac{9}{25}$ / $\Rightarrow \sin x = \sqrt{\frac{9}{25}} \Rightarrow \sin x = \frac{3}{5}$

$x \in \left(0, \frac{\pi}{2}\right)$

Subiectul al II-lea

1. a) $\det A = \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = -1 \cdot 0 - 1 \cdot 0 = 0 - 0 = 0 \Rightarrow$

$\Rightarrow \det A = 0$.

b) $A \cdot (A + I_2) = O_2$, $O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$A \cdot (A + I_2) = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \cdot \left[\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} =$

$= \begin{pmatrix} 0 & -1+1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2 \Rightarrow A \cdot (A + I_2) = O_2$

c) $m = ?$ a.i. $\det B = 0$ unde $B = A \cdot A + m \cdot I_2$

$$A \cdot A = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

$$B = A \cdot A + m \cdot I_2 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} + m \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} =$$

$$= \begin{pmatrix} m+1 & -1 \\ 0 & m \end{pmatrix} \Rightarrow B = \begin{pmatrix} m+1 & -1 \\ 0 & m \end{pmatrix}.$$

$$\det B = 0 \Leftrightarrow \begin{vmatrix} m+1 & -1 \\ 0 & m \end{vmatrix} = 0 \Leftrightarrow m(m+1) = 0 \Rightarrow$$

$\Rightarrow m = 0$ sau $m+1 = 0 \Rightarrow m = 0$ sau $m = -1$.

2. a) $f(-1) = 0$?

$$f(-1) = (-1)^3 + (-1)^2 + 4(-1) + 4 = -1 + 1 - 4 + 4 = 0 \Rightarrow$$

$\Rightarrow f(-1) = 0$.

b) $f(x) : x^2 + 3x + 2 = ?$

$$\begin{array}{r|l} x^3 + x^2 + 4x + 4 & x^2 + 3x + 2 \\ \hline -x^3 - 3x^2 - 2x & x - 2 \\ \hline -2x^2 + 2x + 4 & \\ \underline{2x^2 + 6x + 4} & \\ 8x + 8 & \end{array}$$

Câtul : $x - 2$.

Restul : $8x + 8$.

c) $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_1 x_2} + \frac{1}{x_2 x_3} + \frac{1}{x_1 x_3} = -\frac{3}{4}$?

$$\frac{x_2 x_3}{1} \cdot \frac{1}{x_1} + \frac{x_1 x_3}{1} \cdot \frac{1}{x_2} + \frac{x_1 x_2}{1} \cdot \frac{1}{x_3} + \frac{x_3}{1} \cdot \frac{1}{x_1 x_2} + \frac{x_1}{1} \cdot \frac{1}{x_2 x_3} + \frac{x_2}{1} \cdot \frac{1}{x_1 x_3} = \frac{x_2 x_3 + x_1 x_3 + x_1 x_2 + x_3 + x_1 + x_2}{x_1 x_2 x_3} =$$

N.C $= x_1 x_2 x_3$

$$= \frac{4 - 1}{-4} = \frac{3}{-4} = -\frac{3}{4}$$

$$\begin{cases} x_1+x_2+x_3 = -\frac{b}{a} = -1 \\ x_1x_2 + x_1x_3 + x_2x_3 = \frac{c}{a} = \frac{4}{1} = 4 \\ x_1x_2x_3 = -\frac{d}{a} = -4 \end{cases}$$

Subiectul al III-lea.

1. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 - 12x$.

a) $f'(x) = 3(x-2)(x+2)$, $x \in \mathbb{R}$.

$f'(x) = (x^3 - 12x)' = 3x^2 - 12 = 3(x^2-4) =$

$= 3(x-2)(x+2)$

b) $x = 2$

ec. tangentei într-un punct este:

$y - f(x_0) = f'(x_0)(x - x_0)$

$y - f(2) = f'(2)(x-2)$

$f(2) = 2^3 - 12 \cdot 2 = 8 - 24 = -16$.

$f'(2) = 3(2-2)(2+2) = 3 \cdot 0 \cdot 4 = 0$.

$y + 16 = 0(x-2) \Rightarrow y + 16 = 0 \Rightarrow y = -16$.

c) $-16 \le f(x) \le 16$, $\forall x \in [-2, 2]$.

x	$-\infty$		-2		2		$+\infty$
$f(x)$			16		-16		
$f'(x)$	$+$	$+$	0	$-$	$-$	0	$+$ $+$

$f(-2) = 3(-2-2)(-2+2) = 0$.

$f(2) = 0$.

$f(-2) = (-2)^3 - 12 \cdot (-2) = -8 + 24 = 16$.

$f(2) = 2^3 - 12 \cdot 2 = 8 - 24 = -16$.

$f'(x) \le 0$, $\forall x \in [-2, 2] \Rightarrow f(2) \le f(x) \le f(-2) \Rightarrow -16 \le f(x) \le 16$, $\forall x \in [-2, 2]$

2. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 5x^4 + 3x^2 + 1$.

a) $\int_0^1 (f(x) - 3x^2 - 1)\,dx = 1$?

$$\int_0^1 (f(x) - 3x^2 - 1)\,dx = \int_0^1 (5x^4 + 3x^2 + 1 - 3x^2 - 1)\,dx =$$

$$= \int_0^1 5x^4\,dx = 5 \cdot \frac{x^5}{5}\Big|_0^1 = x^5\Big|_0^1 = 1^5 - 0^5 = 1.$$

b) $A = \int_1^2 |f(x)|\,dx = \int_1^2 (5x^4 + 3x^2 + 1)\,dx =$

$$= \left(\frac{5x^5}{5} + 3 \cdot \frac{x^3}{3} + x\right)\Big|_1^2 = (x^5 + x^3 + x)\Big|_1^2 =$$

$$= 2^5 + 2^3 + 2 - 1^5 - 1^3 - 1 = 32 + 8 + 2 - 1 - 1 - 1 =$$

$$= 42 - 3 = 39$$

c) Fie $F: \mathbb{R} \to \mathbb{R}$ o primitivă a funcției f $\Rightarrow$

$\Rightarrow F'(x) = f(x)$, $x \in \mathbb{R}$.

$F'(x) = 5x^4 + 3x^2 + 1 > 0$, $\forall x \in \mathbb{R}$ $\Rightarrow$

$\Rightarrow \forall F$ este crescătoare pe $\mathbb{R}$.