Simulare

Examenul de bacalaureat național 2017

Proba E. c)

Matematică M_tehnologic

clasa a XII-a

Subiectul I

1. $(2+\sqrt{3})^2+(1-2\sqrt{3})^2=20 \Leftrightarrow 4+\cancel{4\sqrt{3}}+3+1-\cancel{4\sqrt{3}}+12=20 \Leftrightarrow$

$\Leftrightarrow 20=20$ (A)

2. $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^2-3x$

$f(1)\cdot f(2)\cdot f(3)\cdot f(4)=?$

$f(1)=1^2-3\cdot 1=1-3=-2$

$f(2)=2^2-3\cdot 2=4-6=-2$

$f(3)=3^2-3\cdot 3=9-9=0$

$f(4)=4^2-3\cdot 4=16-12=4$

$f(1)\cdot f(2)\cdot f(3)\cdot f(4)=(-2)\cdot(-2)\cdot 0\cdot 4=0$

3. $8^x=4^{2x+1}$, $x=?$, $x\in\mathbb{R}$

$8^x=4^{2x+1} \Leftrightarrow (2^3)^x=(2^2)^{2x+1} \Leftrightarrow 2^{3x}=2^{4x+2}$
$2=2.$ $\Rightarrow$

$\Rightarrow 3x=4x+2 \Leftrightarrow 4x-3x=-2 \Rightarrow x=-2$

4. x = prețul obiectului înainte de scumpire

$^{100)}x+\frac{25}{100}\cdot x=^{100)}250 \Leftrightarrow 100x+25x=25\,000 \Leftrightarrow$

$\Leftrightarrow 125x=25\,000 \Leftrightarrow x=\frac{25000}{125} \Rightarrow x=200$ lei

5. $A(1,5)$, $B(1,1)$, $C(5,5)$, $\triangle ABC$ - isoscel

$$AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}=\sqrt{(1-1)^2+(1-5)^2}=\sqrt{0^2+(-4)^2}=\sqrt{16}=4$$

$$BC=\sqrt{(x_C-x_B)^2+(y_C-y_B)^2}=\sqrt{(5-1)^2+(5-1)^2}=\sqrt{4^2+4^2}=\sqrt{16+16}=$$
$$=\sqrt{32}=4\sqrt{2}$$

$$AC=\sqrt{(x_C-x_A)^2+(y_C-y_A)^2}=\sqrt{(5-1)^2+(5-5)^2}=\sqrt{4^2+0^2}=\sqrt{16}=4$$

$$\left.\begin{array}{l} AB=4 \\ AC=4 \end{array}\right| \Rightarrow AB=AC \Rightarrow \triangle ABC \text{ isoscel}$$

6. $\sin 60^\circ + tg\, 45^\circ = \cos 30^\circ + ctg\, 45^\circ \Leftrightarrow \frac{\sqrt{3}}{2}+1=\frac{\sqrt{3}}{2}+1$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$tg\, 45^\circ = 1$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$ctg\, 45^\circ = 1$

<u>sau</u>

$$\left.\begin{array}{l} \sin 60^\circ = \cos 30^\circ \\ tg\, 45^\circ = ctg\, 45^\circ \end{array}\right| \Rightarrow \sin 60^\circ + tg\, 45^\circ = \cos 30^\circ + ctg\, 45^\circ$$

Subiectul al II - lea

1. $A(x)=\begin{pmatrix} x & 2 \\ x & x \end{pmatrix}$, $x \in \mathbb{R}$

a) $\det(A(3)) = 3$

$\det(A(3)) = \begin{vmatrix} 3 & 2 \\ 3 & 3 \end{vmatrix} = 3 \cdot 3 - 2 \cdot 3 = 9 - 6 = 3$

$A(3) = \begin{pmatrix} 3 & 2 \\ 3 & 3 \end{pmatrix}$

b) $A(2017+x) + A(2017-x) = 2A(2017)$, $(\forall) x \in \mathbb{R}$

$A(2017+x) = \begin{pmatrix} 2017+x & 2 \\ 2017+x & 2017+x \end{pmatrix}$

$A(2017-x) = \begin{pmatrix} 2017-x & 2 \\ 2017-x & 2017-x \end{pmatrix}$

$A(2017) = \begin{pmatrix} 2017 & 2 \\ 2017 & 2017 \end{pmatrix} \Rightarrow 2 \cdot A(2017) = 2\begin{pmatrix} 2017 & 2 \\ 2017 & 2017 \end{pmatrix}$

$A(2017+x) + A(2017-x) = \begin{pmatrix} 2017+x & 2 \\ 2017+x & 2017+x \end{pmatrix} + \begin{pmatrix} 2017-x & 2 \\ 2017-x & 2017-x \end{pmatrix}$

$= \begin{pmatrix} 2017+\cancel{x}+2017-\cancel{x} & 2+2 \\ 2017+\cancel{x}+2017-\cancel{x} & 2017+\cancel{x}+2017-\cancel{x} \end{pmatrix} = \begin{pmatrix} 4034 & 4 \\ 4034 & 4034 \end{pmatrix} =$

$= 2\begin{pmatrix} 2017 & 2 \\ 2017 & 2017 \end{pmatrix} = 2A(2017) \Rightarrow$

$\Rightarrow A(2017+x) + A(2017-x) = 2A(2017)$

c) $m=?$, $m\in\mathbb{R}$, $\det(A(2)+mA(1))=0$

$$A(2)=\begin{pmatrix}2&2\\2&2\end{pmatrix}$$

$$A(1)=\begin{pmatrix}1&2\\1&1\end{pmatrix} \Rightarrow mA(1)=\begin{pmatrix}m&2m\\m&m\end{pmatrix}$$

$$A(2)+mA(1)=\begin{pmatrix}2&2\\2&2\end{pmatrix}+\begin{pmatrix}m&2m\\m&m\end{pmatrix}=\begin{pmatrix}2+m&2+2m\\2+m&2+m\end{pmatrix}$$

$$\det(A(2)+mA(1))=0 \Leftrightarrow \begin{vmatrix}2+m&2+2m\\2+m&2+m\end{vmatrix}=0 \Leftrightarrow$$

$$\Leftrightarrow (2+m)^2-(2+m)(2+2m)=0 \Leftrightarrow (2+m)(\cancel{2}+m-\cancel{2}-2m)=0 \Leftrightarrow$$

$$\Leftrightarrow (2+m)\cdot(-m)=0 \Rightarrow -m=0 \Rightarrow m=0$$

sau

$$2+m=0 \Rightarrow m=-2$$

2. $x*y=2xy+6x+6y+15$, $x,y\in\mathbb{R}$

a) $x*y=2(x+3)(y+3)-3$, $\forall\, x,y\in\mathbb{R}$.

$$2(x+3)(y+3)-3=(2x+6)(y+3)-3=2xy+6x+6y+18-3=$$

$$=2xy+6x+6y+15=x*y. \Rightarrow$$

$$\Rightarrow x*y=2(x+3)(y+3)-3,\ \forall\, x,y\in\mathbb{R}$$

b) $7*98=2017 \Leftrightarrow 2(7+3)(98+3)-3=2017 \Leftrightarrow$

$\Leftrightarrow 2\cdot10\cdot101-3=2017 \Leftrightarrow 20\cdot101-3=2017 \Leftrightarrow$

$\Leftrightarrow 2020-3=2017 \Leftrightarrow 2017=2017$ (A)

c) $x=?$, $x \in \mathbb{R}$, $x * (x+2) = 3$

$x*(x+2)=3 \Leftrightarrow 2(x+3)(x+2+3)-3=3 \Leftrightarrow 2(x+3)(x+5)-3-3=0$

$\Leftrightarrow (2x+6)(x+5)-6=0 \Leftrightarrow 2x^2+10x+6x+30-6=0 \Leftrightarrow$

$\Leftrightarrow 2x^2+16x+24=0 \quad /:2 \Leftrightarrow x^2+8x+12=0$

$x^2+8x+12=0$

$\Delta = b^2-4ac$

$\Delta = 64-4\cdot1\cdot12 = 64-48=16 \Rightarrow \sqrt{\Delta}=\sqrt{16}=4$

$x_{1,2} = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{-8\pm4}{2\cdot1} = \frac{-8\pm4}{2}$ $\rightarrow x_1 = \frac{-8+4}{2} = \frac{-4}{2} = -2$

$x_2 = \frac{-8-4}{2} = \frac{-12}{2} = -6$

Subiectul al III-lea

1. $f:(2,+\infty)\to\mathbb{R}$, $f(x) = x+1+\frac{1}{x-2}$

a) $\lim\limits_{x\to3} \frac{f(x)-f(3)}{x-3} = 0.$

$\lim\limits_{x\to3} \frac{f(x)-f(3)}{x-3} = f'(3)$

$f'(x) = \left(x+1+\frac{1}{x-2}\right)' = (x)'+(1)'+\left(\frac{1}{x-2}\right)' = 1+0+\frac{1'(x-2)-1\cdot(x-2)'}{(x-2)^2} =$

$= 1+\frac{0\cdot(x-2)-1\cdot1}{(x-2)^2} = 1+\frac{-1}{(x-2)^2} = 1-\frac{1}{(x-2)^2}$, $x\in(2,+\infty)$

$f'(3) = 1-\frac{1}{(3-2)^2} = 1-\frac{1}{1^2} = 1-1 = 0 \Rightarrow$

$\Rightarrow \lim\limits_{x\to3}\frac{f(x)-f(3)}{x-3} = 0$, $x\in(2,+\infty)$

b) $y = mx + n$ este ecuația asimptotei oblice, unde

$m = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x}$, iar $n = \lim\limits_{x \to +\infty} \left(f(x) - mx\right)$

$$m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\overset{x-2}{x} + \overset{x-2}{1} + \frac{1}{x-2}}{x} = \lim_{x \to +\infty} \frac{x^2 - 2x + x - 2 + 1}{x(x-2)} =$$

$$= \lim_{x \to +\infty} \frac{x^2 - x - 1}{x^2 - 2x} \overset{\frac{\infty}{\infty}}{=} \lim_{x \to +\infty} \frac{\cancel{x^2}\left(1 - \frac{1}{x} - \frac{1}{x^2}\right)}{\cancel{x^2}\left(1 - \frac{2}{x}\right)} = \frac{1}{1} = 1$$

$m = 1$

$$n = \lim_{x \to +\infty} \left(f(x) - mx\right) = \lim_{x \to +\infty} \left(\cancel{x} + 1 + \frac{1}{x-2} - \cancel{x}\right) =$$

$$= \lim_{x \to +\infty} \left(\overset{x-2}{1} + \frac{1}{x-2}\right) = \lim_{x \to +\infty} \frac{x - 2 + 1}{x - 2} = \lim_{x \to +\infty} \frac{x - 1}{x - 2} \overset{\frac{\infty}{\infty}}{=}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}\left(1 - \frac{1}{x}\right)}{\cancel{x}\left(1 - \frac{2}{x}\right)} = \frac{1}{1} = 1. \Rightarrow n = 1.$$

$$y = mx + n \Rightarrow y = x + 1$$

c) f este convexă $\Leftrightarrow f''(x) \geq 0$

$$f''(x) = \left[1 - \frac{1}{(x-2)^2}\right]' = 1' - \left[\frac{1}{(x-2)^2}\right]' = 0 - \frac{1' \cdot (x-2)^2 - 1 \cdot \left[(x-2)^2\right]'}{(x-2)^4} =$$

$$= -\frac{0 \cdot (x-2)^2 - 2(x-2)(x-2)'}{(x-2)^4} = -\frac{-2(x-2) \cdot 1}{(x-2)^4} = -\frac{-2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}} =$$

$$= \frac{2}{(x-2)^3} > 0 \Rightarrow f''(x) > 0, (\forall)\, x \in (2, +\infty) \Rightarrow$$

($f''(x)$ este strict mai mare ca zero, pt că $f''(2)$ nu este posibilă) ~~(x=2)~~ (în $x=2$ funcția nu e definită)

$\Rightarrow f$ este convexă pe $(2, +\infty)$

2. $f:(0,+\infty)\to\mathbb{R},\ f(x)=1+\ln x$

$F:(0,+\infty)\to\mathbb{R},\ F(x)=x\ln x$

a) $\int_1^e (f(x)-\ln x)\,dx = \int_1^e (1+\cancel{\ln x}-\cancel{\ln x})\,dx = \int_1^e dx =$

$= x\Big|_1^e = e-1$

b) F primitivă a funcției $f \Leftrightarrow F'(x)=f(x)$ și F este derivabilă

$F'(x)=(x\ln x)' = (x)'\ln x + x\cdot(\ln x)' = \ln x + \cancel{x}\cdot\frac{1}{\cancel{x}} =$

$= \ln x + 1 = f(x) \Rightarrow F'(x)=f(x) \Rightarrow$

$\Rightarrow F$ este o primitivă a lui f

c) $\int_1^e f(x)\cdot F(x)\,dx = \frac{e^2}{2}$

$I = \int_1^e f(x)\cdot F(x)\,dx = \int_1^e (1+\ln x)\cdot x\ln x\,dx =$

$= \int_1^e \underbrace{(x\ln x)'}_{F'(x)}\cdot\underbrace{x\ln x}_{F(x)}\,dx = (x\ln x)(x\ln x)\Big|_1^e - \underbrace{\int_1^e \underbrace{(x\ln x)}_{F(x)}\underbrace{(x\ln x)'}_{F'(x)}dx}_{I}$

$I = (x\ln x)^2\Big|_1^e - I \Rightarrow 2I = x^2(\ln x)^2\Big|_1^e \Rightarrow$

$\Rightarrow I = \frac{x^2\ln^2 x}{2}\Big|_1^e \Rightarrow \int_1^e f(x)\cdot F(x)\,dx = \frac{e^2\ln^2 e}{2} - \frac{\ln^2 1}{2} =$

$= \frac{e^2\cdot 1}{2} - \frac{0}{2} = \frac{e^2}{2} \Rightarrow \int_1^e f(x)\cdot F(x)\,dx = \frac{e^2}{2}$