Varianta 01
2016 - M_ tehnologic

Subiectul I.

1. $1-\frac{1}{4}:0,25=0$

$1-0,25:0,25=0.$

$1-1=0$

$0=0.$ (A)

2. $f(-1)\cdot f(1)=?$

$f(x)=x+1$, $f:\mathbb{R}\to\mathbb{R}$.

$f(-1)=-1+1=0$

$f(1)=1+1=2$

$f(-1)\cdot f(1)=0\cdot 2=0$.

3. $\sqrt{2x-3}=5 \overset{()^2}{\Leftrightarrow} 2x-3=5^2 \Leftrightarrow 2x=25+3 \Leftrightarrow 2x=28 \Rightarrow$

C.E $2x-3>0 \Rightarrow 2x>3 \Rightarrow x>\frac{3}{2}$.

$\Rightarrow x=28:2 \Rightarrow x=14$

4. $100+20\%\cdot 100=100+\frac{20}{\cancel{100}}\cdot\cancel{100}=100+20=120$ lei

5. $A(2,4)$, $B(5,4)$

$d(A,B)=?$

$d(A,B)=AB.$

$AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}=\sqrt{(5-2)^2+(4-4)^2}=$

$=\sqrt{3^2+0^2}=\sqrt{3^2}=3$

6. $AB=?$, ABC - $\triangle$ dr. ($m(\sphericalangle A)=90^\circ$), $AC=6$, $\hat{B}=\frac{\pi}{4}$.

$m(\hat{B})=\frac{\pi}{4}=\frac{180^\circ}{4}=45^\circ$
$m(\hat{A})=90^\circ$ $\Big/ \Rightarrow m(\hat{C})=180^\circ-45^\circ-90^\circ=45^\circ$

$m(\hat{B})=m(\hat{C})=45^\circ \Rightarrow \triangle ABC$ este $\triangle$ dr. isoscel $\Rightarrow$
$\Rightarrow AB=AC=6$

Subiectul al II-lea

a) $\det A = \begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix} = -2-2=-4 \Rightarrow \det A = -4$

b) $\det(A-2B) = 0$?

$\det(A-2B) = \begin{vmatrix} 1-2x & 0 \\ 1-2y & 0 \end{vmatrix} = 0-0=0$

$A-2B = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} - 2\begin{pmatrix} x & 1 \\ y & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 2x & 2 \\ 2y & -2 \end{pmatrix} =$

$= \begin{pmatrix} 1-2x & 2-2 \\ 1-2y & -2+2 \end{pmatrix} = \begin{pmatrix} 1-2x & 0 \\ 1-2y & 0 \end{pmatrix}$

c) $x, y = ?$ a.î. $A \cdot B = B \cdot A$

$A \cdot B = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} x & 1 \\ y & -1 \end{pmatrix} = \begin{pmatrix} x+2y & 1-2 \\ x-2y & 1+2 \end{pmatrix} = \begin{pmatrix} x+2y & -1 \\ x-2y & 3 \end{pmatrix}$

$B \cdot A = \begin{pmatrix} x & 1 \\ y & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} x+1 & 2x-2 \\ y-1 & 2y+2 \end{pmatrix}$

$A \cdot B = B \cdot A \Leftrightarrow \begin{pmatrix} x+2y & -1 \\ x-2y & 3 \end{pmatrix} = \begin{pmatrix} x+1 & 2x-2 \\ y-1 & 2y+2 \end{pmatrix} \Rightarrow$

$\Rightarrow \begin{cases} 2x-2=-1 \\ 2y+2=3 \end{cases} \Leftrightarrow \begin{cases} 2x=-1+2 \\ 2y=3-2 \end{cases} \Leftrightarrow \begin{cases} 2x=1 \\ 2y=1 \end{cases} \Rightarrow \begin{cases} x=\frac{1}{2} \\ y=\frac{1}{2} \end{cases}$

2. $x \circ y = xy + 2x + 2y + 2$

a) $1 \circ (-2) = 1 \cdot (-2) + 2 \cdot 1 + 2 \cdot (-2) + 2 = -2 + \cancel{2} - 4 + \cancel{2} = -2 \Rightarrow$

$\Rightarrow 1 \circ (-2) = -2$

b) $x \circ y = (x+2)(y+2) - 2, \forall x, y \in \mathbb{R}$

$(x+2)(y+2)-2 = xy+2x+2y+4-2 = xy+2x+2y+2$

$x \circ y = xy + 2x + 2y + 2$ $\Rightarrow$

$\Rightarrow x \circ y = (x+2)(y+2) - 2, \forall x, y \in \mathbb{R}$

c) $x \circ \frac{1}{x} = x$, $x = ?$, $x \neq 0$.

$$x \circ \frac{1}{x} = \cancel{x} \cdot \frac{1}{\cancel{x}} + 2x + 2 \cdot \frac{1}{x} + 2 = 1 + 2x + \frac{2}{x} + 2 = 2x + \frac{2}{x} + 3$$

$$x \circ \frac{1}{x} = x \Leftrightarrow 2x + \frac{2}{x} + 3 = x \Leftrightarrow 2x - x + \frac{2}{x} + 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow \overset{x}{x} + \frac{2}{x} + \overset{x}{3} = 0 \Rightarrow x^2 + 3x + 2 = 0 \Rightarrow x^2 + x + 2x + 2 = 0 \Leftrightarrow$$

$x \neq 0$.

$$\Leftrightarrow x(x+1) + 2(x+1) = 0 \Leftrightarrow (x+1)(x+2) = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} x+1 = 0 \\ x+2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -1 \\ x_2 = -2 \end{cases}$$

Subiectul al III-lea

1. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3 + x^2 - x + 1$

a) $f'(x) = 3x^2 + 2x - 1$, $x \in \mathbb{R}$?

$(f(x))' = (x^3 + x^2 - x + 1)' = 3x^2 + 2x - 1 + 0 = 3x^2 + 2x - 1$

b) $\lim_{x \to +\infty} \frac{x f'(x)}{f(x)} = 3$?

$\lim_{x \to +\infty} \frac{x f'(x)}{f(x)} = \lim_{x \to +\infty} \frac{x \cdot (3x^2 + 2x - 1)}{x^3 + x^2 - x + 1} =$

$= \lim_{x \to +\infty} \frac{3x^3 + 2x^2 - x}{x^3 + x^2 - x + 1} = \lim_{x \to +\infty} \frac{\cancel{x^3}\left(3 + \underbrace{\frac{3}{x}}_{\to 0} - \underbrace{\frac{1}{x^2}}_{\to 0}\right)}{\cancel{x^3}\left(1 + \underbrace{\frac{1}{x}}_{\to 0} - \underbrace{\frac{1}{x^2}}_{\to 0} + \underbrace{\frac{1}{x^3}}_{\to 0}\right)} = \frac{3}{1} = 3$

c) Pentru a determina abscisele punctelor situate pe graficul funcției f în care tangenta la graficul funcției f este paralelă cu dreapta, trebuie să calculăm: $f'(x) = m$, unde m este panta dreptei $y = 4x + 1$

Dacă $y = 4x + 1 \Rightarrow m = 4$

$f'(x) = 3x^2 + 2x - 1$ $\Big/ \Rightarrow$

$\Rightarrow 3x^2 + 2x - 1 = 4 \Leftrightarrow 3x^2 + 2x - 1 - 4 = 0 \Leftrightarrow 3x^2 + 2x - 5 = 0$

$\Delta = b^2 - 4ac = 4 - 4 \cdot 3 \cdot (-5) = 4 + 60 = 64$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm 8}{6}$ $\Rightarrow x_1 = \frac{-2+8}{6} = \frac{6}{6} = 1$

$x_2 = \frac{-2-8}{6} = \frac{-10}{6} = -\frac{5}{3}$

2. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^5 + x^3 + 2x$

a) $\int_{-1}^{1} (f(x) - x^3 - 2x)\, dx = 0$?

$$\int_{-1}^{1} (f(x) - x^3 - 2x)\, dx = \int_{-1}^{1} (x^5 + \cancel{x^3} + \cancel{2x} - \cancel{x^3} - \cancel{2x})\, dx =$$

$$= \int_{-1}^{1} x^5\, dx = \frac{x^6}{6}\Big|_{-1}^{1} = \frac{1^6}{6} - \frac{(-1)^6}{6} = \frac{1}{6} - \frac{1}{6} = 0$$

b) $\int_{0}^{2} e^x (f(x) - x^5 - x^3 + 1)\, dx = 3e^2 + 1$?

$$\int_{0}^{2} e^x (f(x) - x^5 - x^3 + 1)\, dx = \int_0^2 e^x (\cancel{x^5} + \cancel{x^3} + 2x - \cancel{x^5} - \cancel{x^3} + 1)\, dx =$$

$$= \int_0^2 e^x (2x + 1)\, dx = \int_0^2 2x e^x\, dx + \int_0^2 e^x\, dx =$$

$$= 2\int_0^2 x e^x\, dx + e^x\Big|_0^2 = 2\left(x e^x\Big|_0^2 - \int_0^2 e^x dx\right) + e^x\Big|_0^2 =$$

$f(x) = x$ $f'(x) = 1$

$g'(x) = e^x$ $g(x) = e^x$

$$= 2\left[(2 \cdot e^2 - 0) - e^x\Big|_0^2\right] + (e^2 - e^0) =$$

$$= 2 \cdot 2e^2 - 2(e^2 - e^0) + e^2 - 1 =$$

$$= 4e^2 - 2e^2 + 2 + e^2 - 1 = 3e^2 + 1$$

c) F primitivă a funcției f este convexă pe $\mathbb{R} \Leftrightarrow$
$F''(x) \geq 0, \forall x \in \mathbb{R}$

F este o primitivă a funcției $f \Rightarrow F'(x) = f(x), \forall x \in \mathbb{R}$

$F''(x) = f'(x) = (x^5 + x^3 + 2x)' = 5x^4 + 3x^2 + 2 \geq 0, \forall x \in \mathbb{R}$

$\Rightarrow F$ este convexă pe $\mathbb{R}$.