Examen de bacalaureat 2019
Proba E.c)
Matematică M_tehnologic

Varianta 6

Subiectul I

1. $\sqrt{7}(\sqrt{7}+1)-\sqrt{7}=7 \Leftrightarrow 7+\cancel{\sqrt{7}}-\cancel{\sqrt{7}}=7 \Leftrightarrow 7=7$ (A)

2. $f:\mathbb{R}\to\mathbb{R},\ f(x)=x^2-6x+8$

$G_f \cap Oy \Rightarrow x=0 \Rightarrow y=0^2-6\cdot 0+8 \to y=8 \Rightarrow$

$\Rightarrow \boxed{A(0,8)}$

3. $\log_5(x^2+9)=2 \Leftrightarrow \log_5(x^2+9)=\log_5 5^2 \Rightarrow$

c.e. $x^2+9>0$

$\Rightarrow x^2+9=25 \Leftrightarrow x^2=25-9 \Rightarrow x^2=16 \Leftrightarrow \boxed{\begin{cases} x_1=4 \\ x_2=-4 \end{cases}}$

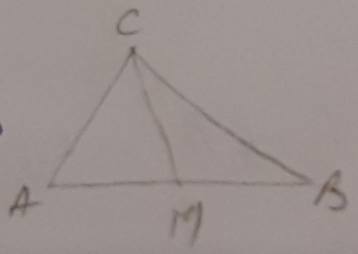
$S=\{-4,4\}$

4. x = prețul produsului înainte de ieftinire

$x-40\%\cdot x=300 \Leftrightarrow x\left(\overset{100)}{1}-\frac{40}{100}\right)=300 \Leftrightarrow x\cdot\frac{6\cancel{0}}{10\cancel{0}}=300 \Leftrightarrow$

$\Leftrightarrow x=\frac{\overset{50}{\cancel{300}}\cdot 10}{\cancel{6}} \Rightarrow \boxed{x=500 \text{ lei}}$

5. $A(3,2),\ B(-3,2),\ C(0,6)$



Fie $M(x_M,y_M)$ mijlocul lui $[AB] \Rightarrow$

$\Rightarrow \begin{cases} x_M=\frac{x_B+x_A}{2}=\frac{-3+3}{2}=0 \\ y_M=\frac{y_B+y_A}{2}=\frac{2+2}{2}=\frac{4}{2}=2 \end{cases} \Rightarrow M(0,2)$

Mediana ΔABC dusă din ~~pun~~ vârful C este $[CM] \Rightarrow$

$\Rightarrow CM=\sqrt{(x_M-x_C)^2+(y_M-y_C)^2}=\sqrt{(0-0)^2+(2-6)^2}=\sqrt{(-4)^2}=\sqrt{16}=4 \Rightarrow$

$\Rightarrow \boxed{CM=4}$

6. $\frac{\sqrt{3}}{2} \cdot \sin 60^\circ - \frac{\sqrt{2}}{2} \cdot \sin 45^\circ = \frac{1}{4} \Leftrightarrow \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{4} \Leftrightarrow$

$\Leftrightarrow \frac{3}{4} - \frac{2}{4} = \frac{1}{4} \Leftrightarrow \frac{1}{4} = \frac{1}{4}$ (A)

Subiectul al II-lea

1. $A = \begin{pmatrix} 6 & -10 \\ 3 & -5 \end{pmatrix}$, $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $M(a) = I_2 + aA$, $a \in \mathbb{R}$

a) $\det A = 0 \Leftrightarrow \begin{vmatrix} 6 & -10 \\ 3 & -5 \end{vmatrix} = 0 \Leftrightarrow 6 \cdot (-5) - (-10) \cdot 3 = 0 \Leftrightarrow$

$\Leftrightarrow -30 + 30 = 0 \Leftrightarrow 0 = 0$ (A)

b) $M(a) \cdot M(b) = M(a+b+ab)$, $(\forall)\, a, b \in \mathbb{R}$

$M(a) = I_2 + aA \Leftrightarrow M(a) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + a \begin{pmatrix} 6 & -10 \\ 3 & -5 \end{pmatrix}$

$$M(a) = \begin{pmatrix} 1+6a & 0-10a \\ 0+3a & 1-5a \end{pmatrix}$$

$$M(a) = \begin{pmatrix} 1+6a & -10a \\ 3a & 1-5a \end{pmatrix}$$

$M(b) = I_2 + bA \Rightarrow M(b) = \begin{pmatrix} 1+6b & -10b \\ 3b & 1-5b \end{pmatrix}$

$M(a+b+ab) = I_2 + (a+b+ab)A \Rightarrow$

$$\Rightarrow M(a+b+ab) = \begin{pmatrix} 1+6(a+b+ab) & -10(a+b+ab) \\ 3(a+b+ab) & 1-5(a+b+ab) \end{pmatrix}$$

$$M(a+b+ab) = M(a)\cdot M(b) \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1+6a & -10a \\ 3a & 1-5a \end{pmatrix} \cdot \begin{pmatrix} 1+6b & -10b \\ 3b & 1-5b \end{pmatrix} = \begin{pmatrix} 1+6a+6b+6ab & -10a-10b-10ab \\ 3a+3b+3ab & 1-5a-5b-5ab \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} (1+6a)(1+6b)+3b\cdot(-10a) & -10b-60ab+(-10a)+50ab \\ 3a+18ab+3b-15ab & -30ab+(1-5a)(1-5b) \end{pmatrix} =$$

$$= \begin{pmatrix} 1+6a+6b+6ab & -10a-10b-10ab \\ 3a+3b+3ab & 1-5a-5b-5ab \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1+6b+6a+36ab-30ab & -10a-10b-10ab \\ 3a+3b+3ab & -30ab+1-5b-5a+25ab \end{pmatrix} =$$

$$= \begin{pmatrix} 1+6a+6b+6ab & -10a-10b-10ab \\ 3a+3b+3ab & 1-5a-5b-5ab \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1+6a+6b+6ab & -10a-10b-10ab \\ 3a+3b+3ab & 1-5a-5b-5ab \end{pmatrix} =$$

$$= \begin{pmatrix} 1+6a+6b+6ab & -10a-10b-10ab \\ 3a+3b+3ab & 1-5a-5b-5ab \end{pmatrix} \quad (A) \Rightarrow$$

$$\Rightarrow M(a)\cdot M(b) = M(a+b+ab), (\forall) a,b \in \mathbb{R}$$

c) $a = ?$

$$M(1) + M(2) + \dots + M(2019) = 2019\, M(a)$$

$$M(1) = I_2 + 1 \cdot A = I_2 + A$$

$$M(2) = I_2 + 2 \cdot A$$

$$M(3) = I_2 + 3A$$

$$\vdots$$

$$M(2019) = I_2 + 2019A.$$

$$M(1) + M(2) + \dots + M(2019) = 2019 \cdot I_2 + (1+2+3+\dots+2019) \cdot A =$$

$$= 2019 \cdot I_2 + \frac{2019 \cdot (2019+1)}{2} \cdot A =$$

$$= 2019 \cdot I_2 + \frac{2019 \cdot \overset{1010}{\cancel{2020}}}{\cancel{2}} \cdot A =$$

$$= 2019 \cdot I_2 + 2019 \cdot 1010 \cdot A$$

$$M(1) + M(2) + \dots + M(2019) = 2019\, M(a) \Leftrightarrow$$

$$\Leftrightarrow 2019 \cdot I_2 + 2019 \cdot 1010 \cdot A = 2019\, M(a) \quad \Big| : 2019 \Leftrightarrow$$

$$\Leftrightarrow I_2 + 1010 \cdot A = M(a) \Leftrightarrow$$

$$\Leftrightarrow \cancel{I_2} + 1010 \cdot A = \cancel{I_2} + aA \Leftrightarrow 1010 \cdot A = a \cdot A \Rightarrow a = 1010$$

2. $f = mX^3 + 2X^2 - mX - 2 \quad , m \in \mathbb{R}^*$

a) $f(1) = 0 \Leftrightarrow m \cdot 1^3 + 2 \cdot 1^2 - m \cdot 1 - 2 = 0 \Leftrightarrow \cancel{m} + \cancel{2} - \cancel{m} - \cancel{2} = 0 \Leftrightarrow$

$\Leftrightarrow 0 = 0 \quad (A) \quad , (\forall)\ m \in \mathbb{R}^*$

b) $m = 3 \Rightarrow f = 3X^3 + 2X^2 - 3X - 2 = X^2(3X+2) - (3X+2) =$

$= (3X+2)(X^2-1) = (3X+2)(X-1)(X+1) \Rightarrow$

$\Rightarrow f = (3X+2)(X-1)(X+1)$

$$f(x) = 0 \Leftrightarrow (3x+2)(x-1)(x+1) = 0 \Rightarrow \left| \begin{array}{l} 3x+2 = 0 \\ x-1 = 0 \\ x+1 = 0. \end{array} \right. \Rightarrow$$

$$\Rightarrow \left| \begin{array}{l} x_1 = -\frac{2}{3} \\ x_2 = 1 \\ x_3 = -1 \end{array} \right.$$

c) $m = ?$ a.î. $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = -4 \quad ; f = mX^3 + 2X^2 - mX - 2$

Relațiile lui Viete: $\left| \begin{array}{l} x_1 + x_2 + x_3 = -\frac{b}{a} = -\frac{2}{m} \\ x_1x_2 + x_2x_3 + x_1x_3 = \frac{c}{a} = -\frac{\cancel{m}}{\cancel{m}} = -1 \\ x_1x_2x_3 = -\frac{d}{a} = -\frac{-2}{m} = \frac{2}{m} \end{array} \right.$

$$\overset{x_2x_3}{\frac{1}{x_1}} + \overset{x_1x_3}{\frac{1}{x_2}} + \overset{x_1x_2}{\frac{1}{x_3}} = -4 \Leftrightarrow \frac{x_2x_3 + x_1x_3 + x_1x_2}{x_1x_2x_3} = -4 \Leftrightarrow \frac{-1}{\frac{2}{m}} = -4 \Leftrightarrow$$

$$\Leftrightarrow -\frac{m}{2} = -4 \ |\cdot(-1) \Leftrightarrow \frac{m}{2} = 4 \Leftrightarrow m = 8$$

Subiectul al III-lea

1. $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^3-3x+5$

a) $f'(x)=3(x-1)(x+1)$, $x\in\mathbb{R}$.

$f'(x)=(x^3-3x+5)'=3x^2-3=3(x^2-1)=3(x-1)(x+1)$

$\Rightarrow f'(x)=3(x-1)(x+1)$ (A)

b) f convexă $\Leftrightarrow f''(x)\geq 0$

$f''(x)=(3x^2-3)'=6x$

$f''(x)=0 \Rightarrow 6x=0 \Rightarrow x=0$

x	$-\infty$		0		$+\infty$
$f''(x)$	$-\ -$	$-\ -$	0	$+\ +$	$+\ +$
$f(x)$		$\searrow$	5	$\nearrow$	

$f(0)=0^3-3\cdot 0+5=5$

$f''(x)\leq 0 \Rightarrow f(x)$ descrescătoare pe $(-\infty;0]$

$f''(x)\geq 0 \Rightarrow f(x)$ crescătoare pe $[0,+\infty)$ $\Rightarrow$

$\Rightarrow f$ convexă pe $[0,+\infty)$

c) $f(x) \le 7, (\forall) x \in (-\infty, 1]$

x	$-\infty$			-1				1			$+\infty$
$f'(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
$f(x)$		$\nearrow$		7		$\searrow$		3		$\nearrow$	
				max				min			

$$f'(x)=0 \Leftrightarrow 3(x-1)(x+1)=0 \Rightarrow \begin{cases} x-1=0 \\ x+1=0 \end{cases} \Rightarrow \begin{cases} x_1=1 \\ x_2=-1 \end{cases}$$

$$f(-1)=(-1)^3-3(-1)+5=5=-1+3+5=7$$

$$f(1)=1^3-3\cdot 1+5=1-3+5=3$$

$f(x)$ este crescătoare pe intervalele $(-\infty;-1]$ și $[1,+\infty)$

$f(x)$ este descrescătoare pe intervalul $[-1,1]$ } $\Rightarrow$

$$\Rightarrow f(x) \le f(-1) \Rightarrow f(x) \le 7, (\forall) x \in (-\infty, 1]$$

2. $f:\mathbb{R}\to\mathbb{R}, f(x)=\sqrt{3x^2+6x+7}$

a) $\int_0^1 f^2(x)\,dx=11 \Leftrightarrow \int_0^1 \left(\sqrt{3x^2+6x+7}\right)^2 dx = 11 \Leftrightarrow$

$$\Leftrightarrow \int_0^1 (3x^2+6x+7)\,dx = 11 \Leftrightarrow \int_0^1 3x^2 dx + \int_0^1 6x\,dx + \int_0^1 7\,dx = 11 \Leftrightarrow$$

$$\Leftrightarrow \left(\cancel{3}\frac{x^3}{\cancel{3}_1} + \overset{3}{\cancel{6}}\frac{x^2}{\cancel{2}_1} + 7x\right)\Big|_0^1 = 11 \Leftrightarrow \left(x^3+3x^2+7x\right)\Big|_0^1 = 11 \Leftrightarrow$$

$$\Leftrightarrow 1^3+3\cdot 1^2+7\cdot 1 - 0^3+3\cdot 0^2+7\cdot 0 = 11 \Leftrightarrow 1+3+7-0=11 \Leftrightarrow 11=11\ (A)$$

b) $\int_{-1}^{1} \frac{x+1}{f(x)} dx = \int_{-1}^{1} \frac{x+1}{\sqrt{3x^2+6x+7}} dx = \frac{1}{6} \int_{-1}^{1} \frac{6(x+1)}{\sqrt{3x^2+6x+7}} dx =$

$(3x^2+6x+7)' = 6x+6$

$= \frac{1}{6} \int_{-1}^{1} \frac{(3x^2+6x+7)'}{\sqrt{3x^2+6x+7}} dx = \frac{1}{\underset{3}{\cancel{6}}} \cdot \overset{1}{\cancel{2}} \sqrt{3x^2+6x+7} \Big|_{-1}^{1} =$

Aplicăm: $\int \frac{u'}{\sqrt{u}} du = 2\sqrt{u} + C.$

$= \frac{\sqrt{3x^2+6x+7}}{3} \Big|_{-1}^{1} = \frac{\sqrt{3 \cdot 1^2 + 6 \cdot 1 + 7}}{3} - \frac{\sqrt{3 \cdot (-1)^2 + 6 \cdot (-1) + 7}}{3} =$

$= \frac{\sqrt{3+6+7}}{3} - \frac{\sqrt{3-6+7}}{3} = \frac{\sqrt{16}}{3} - \frac{\sqrt{4}}{3} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3} \Rightarrow$

$\Rightarrow \int_{-1}^{1} \frac{x+1}{f(x)} dx = \frac{2}{3}$

c) $A = \int_{0}^{a} |f(x)| dx = \int_{0}^{a} \left|\sqrt{3x^2+6x+7}\right| dx = \left| \int_{0}^{a} \sqrt{3x^2+6x+7} \, dx \right| \geq$

$\geq \int_{0}^{a} |\sqrt{7}| dx. \Rightarrow \int_{0}^{a} \sqrt{3x^2+6x+7} \, dx \geq \int_{0}^{a} \sqrt{7} \, dx \Rightarrow$

$\sqrt{3x^2+6x+7} \geq \sqrt{7}$, $(\forall)\ a \in (0, +\infty)$

$\Rightarrow \int_{0}^{a} \sqrt{3x^2+6x+7} \, dx \geq \sqrt{7} \cdot x \Big|_{0}^{a} \Rightarrow \int_{0}^{a} \sqrt{3x^2+6x+7} \, dx \geq a\sqrt{7}$, $(\forall) a \in (0,$