Examenul de bacalaureat național 2019
Proba E. c)
Matematică M_tehnologic
= Varianta 8 =

Subiectul I

1. $\left(\frac{1}{1}^{12} - \frac{1}{3}^{4} + \frac{1}{4}^{3}\right) : \left(\frac{1}{1}^{12} - \frac{1}{12}\right) = 1 \Leftrightarrow \frac{12-4+3}{12} : \frac{12-1}{12} = 1 \Leftrightarrow$

$\Leftrightarrow \frac{\cancel{11}^{1}}{\cancel{12}_{1}} \cdot \frac{\cancel{12}^{1}}{\cancel{11}_{1}} = 1 \Leftrightarrow 1 = 1$ (A)

2. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + 4$

$f(-2) + f(2) = 4 f(0) \Leftrightarrow 8 + 8 = 4 \cdot 4 \Leftrightarrow 16 = 16$ (A)

$f(-2) = (-2)^2 + 4 = 4 + 4 = 8$

$f(2) = 2^2 + 4 = 4 + 4 = 8$

$f(0) = 0^2 + 4 = 4$

3. $\log_8 (x^2 - 27) = \log_8 (x-3)^2 \Leftrightarrow x^2 - 27 = (x-3)^2 \Leftrightarrow$

c.e. $x^2 - 27 > 0$
$(x-3)^2 > 0$

$\Leftrightarrow \cancel{x^2} - 27 = \cancel{x^2} - 6x + 9 \Leftrightarrow 6x = 27 + 9 \Leftrightarrow 6x = 36 \Leftrightarrow$

$\Leftrightarrow x = 36 : 6 \Rightarrow x = 6 \in \mathbb{R}$ (verifică egalitatea)

4. $P = ?$

$M = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$

nr. cazurilor favorabile $= \{10, 12, 14, 16, 18\} = 5$

nr. cazurilor posibile $= 10$

$P = \frac{5^{(5}}{10} = \frac{1}{2}$

5. $A(4,3)$, $B(8,3)$

$C(x_C, y_C) = ?$

B mijlocul segmentului AC $\Rightarrow \begin{cases} x_B = \dfrac{x_C + x_A}{2} \\ y_B = \dfrac{y_C + y_A}{2} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 8 = \dfrac{x_C + 4}{2} \\ 3 = \dfrac{y_C + 3}{2} \end{cases} \Leftrightarrow \begin{cases} x_C + 4 = 16 \\ y_C + 3 = 2 \cdot 3 \end{cases} \Leftrightarrow \begin{cases} x_C = 16 - 4 \\ y_C = 6 - 3 \end{cases} \Leftrightarrow \begin{cases} x_C = 12 \\ y_C = 3 \end{cases} \Rightarrow$

$\Rightarrow C(12, 3)$

6. $\cos^2 30^\circ + \sin^2 60^\circ - 2\cos 30^\circ \cdot \sin 60^\circ = 0 \Leftrightarrow$

$\Leftrightarrow (\cos 30^\circ - \sin 60^\circ)^2 = 0 \Leftrightarrow \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)^2 = 0 \Rightarrow 0 = 0$ (A)

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

Subiectul al II-lea

1. $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$; $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; $A(a) = \begin{pmatrix} a & 1 \\ 3 & 2 \end{pmatrix}$, $a \in \mathbb{R}$

a) $\det M = 3 \Leftrightarrow \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 \Leftrightarrow 4 - 1 = 3 \Leftrightarrow 3 = 3$ (A)

b) $a = ?$

$A(a) \cdot A(a) = 4A(a) - I_2$

$A(a) \cdot A(a) = \begin{pmatrix} a & 1 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} a & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} a^2+3 & a+2 \\ 3a+6 & 3+4 \end{pmatrix} = \begin{pmatrix} a^2+3 & a+2 \\ 3a+6 & 7 \end{pmatrix}$

$4 \cdot A(a) = 4 \cdot \begin{pmatrix} a & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4a & 4 \\ 12 & 8 \end{pmatrix}$

$A(a) \cdot A(a) = 4A(a) - I_2 \Leftrightarrow \begin{pmatrix} a^2+3 & a+2 \\ 3a+6 & 7 \end{pmatrix} = \begin{pmatrix} 4a & 4 \\ 12 & 8 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Leftrightarrow$

$\Leftrightarrow \begin{pmatrix} a^2+3 & a+2 \\ 3a+6 & 7 \end{pmatrix} = \begin{pmatrix} 4a-1 & 4 \\ 12 & 8-1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} a^2+3 & a+2 \\ 3a+6 & 7 \end{pmatrix} = \begin{pmatrix} 4a-1 & 4 \\ 12 & 7 \end{pmatrix} \Rightarrow$

$\Rightarrow a + 2 = 4 \Rightarrow a = 4 - 2 \Rightarrow \boxed{a = 2} \in \mathbb{R}$

c) $a = ?$, $\det(aA(a) + M) = 0$

$$aA(a) + M = a \cdot \begin{pmatrix} a & 1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} a^2 & a \\ 3a & 2a \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} a^2 + 2 & a+1 \\ 3a+1 & 2a+2 \end{pmatrix}$$

$$\det(aA(a) + M) = 0 \Leftrightarrow \begin{vmatrix} a^2 + 2 & a+1 \\ 3a+1 & 2a+2 \end{vmatrix} = 0 \Leftrightarrow$$

$\Leftrightarrow (a^2+2)(2a+2) - (3a+1) \cdot (a+1) = 0 \Leftrightarrow$

$\Leftrightarrow 2a^3 + 2a^2 + \cancel{4a} + 4 - 3a^2 - \cancel{3a} - \cancel{a} - 1 = 0 \Leftrightarrow$

$\Leftrightarrow 2a^3 - a^2 + 3 = 0 \Leftrightarrow 2a^3 + 2 - a^2 + 1 = 0 \Leftrightarrow$

$\Leftrightarrow 2(a^3 + 1) - (a^2 - 1) = 0 \Leftrightarrow 2(a+1)(a^2 - a + 1) - (a-1)(a+1) = 0 \Leftrightarrow$

$\Leftrightarrow (a+1)(2a^2 - 2a + 2 - a + 1) = 0 \Leftrightarrow (a+1)(2a^2 - 3a + 3) = 0 \Rightarrow$

$\Rightarrow a + 1 = 0$ și $2a^2 - 3a + 3 = 0$.

I $a + 1 = 0 \Rightarrow a = -1 \in \mathbb{R}$.

II $2a^2 - 3a + 3 = 0$

$\Delta = b^2 - 4ac = (-3)^2 - 4 \cdot 2 \cdot 3 = 9 - 24 = -15 < 0 \Rightarrow$

nu convine.

Deci, $a = -1$.

2. $f = X^3 - 4X^2 + mX + 2$, $m \in \mathbb{R}$

a) $f(2) = 2m - 6$, $(\forall)$ $m \in \mathbb{R}$

$f(2) = 2m - 6 \Leftrightarrow 2^3 - 4 \cdot 2^2 + m \cdot 2 + 2 = 2m - 6 \Leftrightarrow$

$\Leftrightarrow 8 - 16 + 2m + 2 = 2m - 6 \Leftrightarrow 2m - 6 = 2m - 6$, $(\forall)$ $m \in \mathbb{R}$

b) $E = x_1^2 x_2 x_3 + x_1 x_2^2 x_3 + x_1 x_2 x_3^2 \in \mathbb{Z}$, $(\forall)$ $m \in \mathbb{R}$

$E = x_1^2 x_2 x_3 + x_1 x_2^2 x_3 + x_1 x_2 x_3^2 = x_1 x_2 x_3 \cdot (x_1 + x_2 + x_3) =$

$= -2 \cdot 4 = -8 \in \mathbb{Z}$ (A), $(\forall)$ $m \in \mathbb{R}$.

Știm că:
Relațiile lui Viète

$$\begin{cases} x_1 + x_2 + x_3 = -\frac{b}{a} = -\frac{-4}{1} = 4 \\ x_1 x_2 + x_1 x_3 + x_2 x_3 = \frac{c}{a} = \frac{m}{1} = m \\ x_1 x_2 x_3 = -\frac{d}{a} = -\frac{2}{1} = -2 \end{cases}$$

c) $m = 3 \Rightarrow f = X^3 - 4X^2 + 3X + 2$

$X^3 - 4X^2 + 3X + 2 = 0$

	X^3	X^2	X^1	X^0
	1	-4	3	2
2	1	-2	-1	0

$\Rightarrow x_1 = 2$

$X^2 - 2X - 1 = 0$

$\Delta = b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot (-1) = 4 + 4 = 8$

$\sqrt{\Delta} = \sqrt{8} = 2\sqrt{2}$

$x_{2,3} = \frac{-b \pm \sqrt{\Delta}}{2a}$

$\Rightarrow x_2 = \frac{2 + 2\sqrt{2}}{2} \Rightarrow x_2 = 1 + \sqrt{2}$

$\Rightarrow x_3 = \frac{2 - 2\sqrt{2}}{2} = \frac{\cancel{2}(1 - \sqrt{2})}{\cancel{2}} = 1 - \sqrt{2}$

$x_3 = 1 - \sqrt{2}$

Rădăcinile polinomului sunt: $1 - \sqrt{2}$; $1 + \sqrt{2}$ și 2

Subiectul al III-lea

1. $f:\mathbb{R}\to\mathbb{R},\ f(x)=7x^3-5x^2+x+1$

a) $f'(x)=(3x-1)(7x-1),\ x\in\mathbb{R}$

$$f'(x)=(7x^3-5x^2+x+1)'=21x^2-10x+1=21x^2-7x-3x+1=$$

$$=7x(3x-1)-(3x-1)=(3x-1)(7x-1)\Rightarrow$$

$$\Rightarrow f'(x)=(3x-1)(7x-1)\ (A),\ x\in\mathbb{R}$$

b) $$\lim_{x\to+\infty}\frac{x f'(x)}{f(x)}=\lim_{x\to+\infty}\frac{x\cdot(21x^2-10x+1)}{7x^3-5x^2+x+1}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^3}\left(21-\underbrace{\frac{10}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}\right)}{\cancel{x^3}\left(7-\underbrace{\frac{5}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}+\underbrace{\frac{1}{x^3}}_{\to 0}\right)}=\frac{21}{7}=3\Rightarrow$$

$$\Rightarrow\lim_{x\to+\infty}\frac{x f'(x)}{f(x)}=3$$

c) $f(x)\le\frac{52}{49},\ (\forall)\ x\in\left(-\infty;\frac{1}{3}\right]$

x	$-\infty$		$\frac{1}{7}$		$\frac{1}{3}$		$+\infty$
$f'(x)$	+ +		0	− −	0	+ +	
$f(x)$	$\nearrow$		$\frac{52}{49}$ max.	$\searrow$	$\frac{28}{27}$ min	$\nearrow$	

$f'(x)=0\Leftrightarrow(3x-1)(7x-1)=0\Rightarrow 3x-1=0$ și $7x-1=0\Rightarrow$

$\Rightarrow x_1=\frac{1}{3}$ și $x_2=\frac{1}{7}$

$$f\left(\frac{1}{3}\right)=7\cdot\left(\frac{1}{3}\right)^3-5\cdot\left(\frac{1}{3}\right)^2+\frac{1}{3}+1=\frac{7}{27}-{}^{3)}\frac{5}{9}+{}^{9)}\frac{1}{3}+{}^{27)}\frac{1}{1}=\frac{7-15+9+27}{27}=\frac{28}{27}$$

$$f\left(\frac{1}{7}\right) = 7\cdot\left(\frac{1}{7}\right)^3 - 5\left(\frac{1}{7}\right)^2 + \frac{1}{7} + 1 = \cancel{7}^{1}\cdot\frac{1}{7^{\cancel{3}2}} - \frac{5}{49} + \frac{{}^{7)}1}{7} + \frac{{}^{49)}1}{1} =$$

$$= \frac{1-5+7+49}{49} = \frac{52}{49}$$

$f(x)$ crescătoare pe $\left(-\infty; \frac{1}{7}\right] \cup \left[\frac{1}{3}, +\infty\right)$

$f(x)$ descrescătoare pe $\left[\frac{1}{7}; \frac{1}{3}\right]$

$$\Rightarrow f(x) \leq f\left(\frac{1}{7}\right) \Rightarrow f(x) \leq \frac{52}{49}, \ (\forall)\ x \in \left(-\infty; \frac{1}{3}\right]$$

2. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} x^2+8x-2, & x \in (-\infty, 0] \\ x-2, & x \in (0, +\infty) \end{cases}$

a) $\int_1^2 f(x)\,dx = -\frac{1}{2} \Leftrightarrow \int_1^2 (x-2)\,dx = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \int_1^2 x\,dx - 2\int_1^2 dx = -\frac{1}{2} \Leftrightarrow \left(\frac{x^2}{2} - 2\cdot x\right)\Big|_1^2 = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \frac{2^{\cancel{2}1}}{\cancel{2}_1} - 2\cdot 2 - \frac{1^2}{2} + 2\cdot 1 = -\frac{1}{2} \Leftrightarrow \cancel{2} - \cancel{4} - \frac{1}{2} + \cancel{2} = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow -\frac{1}{2} = -\frac{1}{2}$ (A)

b) f admite primitive pe $\mathbb{R} \Leftrightarrow f$ continuă pe $\mathbb{R}$

$\lim\limits_{\substack{x \to 0 \\ x<0}} f(x) = \lim\limits_{\substack{x \to 0 \\ x<0}} (x^2+8x-2) = 0+0-2 = -2$

$\lim\limits_{\substack{x \to 0 \\ x>0}} f(x) = \lim\limits_{\substack{x \to 0 \\ x>0}} (x-2) = 0-2 = -2$

$f(0) = 0^2 + 8\cdot 0 - 2 = -2$

$\Rightarrow f$ continuă pe $(-\infty; 0]$ și $(0, +\infty) \Rightarrow$

$\Rightarrow f$ continuă pe $\mathbb{R} \Rightarrow$

$\Rightarrow f$ admite primitive pe $\mathbb{R}$.

c) $A = \int_{-1}^{0} |f(x)|\, dx = \int_{-1}^{0} |x^2 + 8x - 2|\, dx =$

$= \left| \int_{-1}^{0} x^2\, dx + 8\int_{-1}^{0} x\, dx - 2\int_{-1}^{0} dx \right| =$

$= \left| \left(\frac{x^3}{3} + \cancel{8}^{4} \cdot \frac{x^2}{\cancel{2}_1} - 2x \right) \Big|_{-1}^{0} \right| = \left| \frac{(0^3)}{3} + 4 \cdot 0^2 - 2 \cdot 0 - \right.$

$\left. - \left[\frac{(-1)^3}{3} + 4(-1)^2 - 2 \cdot (-1) \right] \right| = \left| 0 - \left(\frac{-1}{3} + 4 + 2 \right) \right| =$

$= \left| \frac{1}{3} - \overset{3}{\cancel{6}} \right| = \left| \frac{1-18}{3} \right| = \left| -\frac{17}{3} \right| = \frac{17}{3}$