

M. Tehnologie
Varianta 2
2017

Subiectul I

1. $(2 + \frac{1}{2}) \cdot \frac{4}{5} = 2$

$$(\frac{2}{2} + \frac{1}{2}) \cdot \frac{4}{5} = \frac{4+1}{2} \cdot \frac{4}{5} = \frac{5}{2} \cdot \frac{4}{5} = 2$$

2. $\frac{x_1 + x_2 - 1}{x_1 x_2} = 1$

$$x^2 - 4x + 3 = 0$$

Dim relatii lui Viète:

$$\left. \begin{aligned} x_1 + x_2 &= -\frac{b}{a} = -\frac{-4}{1} = 4 \\ x_1 x_2 &= \frac{c}{a} = 3 \end{aligned} \right\}$$

$$\frac{x_1 + x_2 - 1}{x_1 x_2} = \frac{4 - 1}{3} = \frac{3}{3} = 1$$

3. $2^{x+1} = 8 \Leftrightarrow 2^{x+1} = 2^3 \Rightarrow x+1 = 3 \Rightarrow x = 3-1 = 2$

4. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$M_4 = \{1 \cdot 4, 2 \cdot 4\} = \{4, 8\}$$

$$P = \frac{\text{nr. cazurilor favorabile}}{\text{nr. cazurilor posibile}} = \frac{2}{9}$$

5. $A(0, 3); B(4, 0)$, $P_{\Delta OAB} = OA + OB + AB$

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(4-0)^2 + (0-3)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$O(0, 0)$$

$$OA = \sqrt{(x_A - x_O)^2 + (y_A - y_O)^2} = \sqrt{(0-0)^2 + (3-0)^2} = \sqrt{0+9} = \sqrt{9} = 3$$

$$OB = \sqrt{4^2 - 0^2} = \sqrt{16} = 4$$

$$P_{\Delta OAB} = 3 + 4 + 5 = 12$$

$$6. \sin^2 150^\circ + \sin^2 60^\circ = 1$$

$$\sin 150^\circ = \sin(180^\circ - 150^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin^2 150^\circ + \sin^2 60^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

Subiectul al II-lea

$$1. A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix}, \quad a \in \mathbb{R}$$

$$a) \det A = 5$$

$$\det A = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$$

$$b) a = ?, \quad B \cdot B = 2B \Leftrightarrow \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix} = 2 \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} 1+1 & 1+a \\ 1+a & 1+a^2 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2a \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 & 1+a \\ 1+a & 1+a^2 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2a \end{pmatrix} \Leftrightarrow$$

$$\Rightarrow \begin{cases} 1+a = 2 \\ 1+a^2 = 2a \end{cases} \Rightarrow \begin{cases} a = 2-1 \\ a^2 - 2a + 1 = 0 \end{cases} \Rightarrow \begin{cases} a = 1 \\ (a-1)^2 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow a = 1$$

$$c) \det(A \cdot B - B \cdot A) \geq 0, \quad (\forall) a \in \mathbb{R}$$

$$A \cdot B = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix} = \begin{pmatrix} 3+2 & 3+2a \\ 2+3 & 2+3a \end{pmatrix} = \begin{pmatrix} 5 & 3+2a \\ 5 & 2+3a \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 1 & 1 \\ 1 & a \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3+2 & 2+3 \\ 3+2a & 2+3a \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ 3+2a & 2+3a \end{pmatrix}$$

$$A \cdot B - B \cdot A = \begin{pmatrix} 5 & 3+2a \\ 5 & 2+3a \end{pmatrix} - \begin{pmatrix} 5 & 5 \\ 3+2a & 2+3a \end{pmatrix} = \begin{pmatrix} 0 & 2a-2 \\ 2-2a & 0 \end{pmatrix}$$

$$\det(A \cdot B - B \cdot A) = \begin{vmatrix} 0 & 2a-2 \\ 2-2a & 0 \end{vmatrix} = 0 - (2-2a)(2a-2) =$$

$$= -(4a-4-4a^2+4a) = 4a^2-8a+4 = 4(a^2-2a+1) =$$

$$= 4(a-1)^2 \geq 0, (\forall) a \in \mathbb{R}$$

$$2. \quad x \circ y = xy - 3x - 3y + 12$$

$$a) \quad 1 \circ 3 = 3$$

$$1 \circ 3 = 1 \cdot 3 - 3 \cdot 1 - 3 \cdot 3 + 12 = \cancel{3} \cdot \cancel{3} - 9 + 12 = 3$$

$$b) \quad x \circ y = (x-3)(y-3) + 3 = xy - 3x - 3y + 9 + 3 = xy - 3x - 3y + 12$$

$$c) \quad (x \circ x) \circ x = 3$$

$$x \circ x = (x-3)(x-3) + 3 = (x-3)^2 + 3$$

$$(x \circ x) \circ x = \left[(x-3)^2 + \cancel{x} - \cancel{x} \right] (x-3) + 3 = (x-3)^2 (x-3) + 3 =$$

$$= (x-3)^3 + 3$$

$$(x-3)^3 + 3 = 3 \Leftrightarrow (x-3)^3 = 3-3 \Leftrightarrow (x-3)^3 = 0 \Rightarrow$$

$$\Rightarrow x-3 = 0 \Leftrightarrow x = 3$$

Subiectul al III-lea

$$1. \quad f: \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = x^3 + 6x + 2$$

$$a) \quad f'(x) = 3(x^2 + 2), \quad x \in \mathbb{R}$$

$$f'(x) = (x^3 + 6x + 2)' = 3x^2 + 6 = 3(x^2 + 2)$$

$$b) \quad \lim_{x \rightarrow 0} \frac{f'(x)}{x+2} = 3 \Leftrightarrow \lim_{x \rightarrow 0} \frac{3(x^2+2)}{x+2} = \frac{3(0+2)}{0+2} =$$

$$= \frac{3 \cdot 2}{2} = 3$$

$$c) -5 \leq f(x) \leq 9, x \in [-1, 1]$$

$$-5 \leq (x^3 + 6x + 2) \leq 9$$

Deci $x \in [-1, 1]$

$$f'(x) = 3(x^2 + 2) > 0 \Rightarrow f(x) \text{ este crescătoare pe } [-1, 1]$$

$$f(-1) = (-1)^3 - 6 + 2 = -1 - 4 = -5$$

$$f(1) = 1^3 + 6 + 2 = 1 + 6 + 2 = 9$$

$$\Rightarrow -5 \leq f(x) \leq 9, (\forall) x \in [-1, 1]$$

$$2. f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 4x^3 - x$$

$$a) \int_0^1 (f(x) + x) dx = \int_0^1 (4x^3 - x + x) dx = \int_0^1 4x^3 dx =$$

$$= 4 \cdot \frac{x^4}{4} \Big|_0^1 = x^4 \Big|_0^1 = 1^4 - 0^4 = 1$$

$$b) \int_0^1 (4x^3 - f(x)) e^x dx = \int_0^1 (4x^3 - 4x^3 + x) e^x dx =$$

$$= \int_0^1 x e^x dx = x e^x \Big|_0^1 - \int_0^1 e^x dx = x e^x \Big|_0^1 - e^x \Big|_0^1 =$$

$$f(x) = x \quad f'(x) = 1$$

$$g'(x) = e^x \quad g(x) = e^x$$

$$= 1 \cdot e^1 - 0 - e^1 + e^0 = \cancel{e} - \cancel{e} + 1 = 1$$

$$c) \text{ Aria} = \int_1^3 |f(x)| dx = \int_1^3 (4x^3 - x) dx = \int_1^3 4x^3 dx -$$

$$\int_1^3 x dx = 4 \cdot \frac{x^4}{4} \Big|_1^3 - \frac{x^2}{2} \Big|_1^3 = x^4 \Big|_1^3 - \left(\frac{3^2}{2} - \frac{1}{2} \right) =$$

$$= 3^4 - 1 - \left(\frac{9-1}{2} \right) = 81 - 1 - \frac{8}{2} = 80 - 4 = 76$$