

Lectia 3

Operatii cu multimii

1. a) $A = \{a, b, c, f, h\}$

$$B = \{a, d, e, f, h\}$$

$$C = \{e, g, h\}$$

b) $\Delta = \{x \mid x \in A \text{ și } x \in B\} = \{a, f, h\}$

$$E = \{x \mid x \in B \text{ și } x \in C\} = \{e, h\}$$

$$M = \{x \mid x \in A \text{ și } x \notin B\} = \{b, c\}$$

2. a) $\{3\} \cap \{a\} = \emptyset$ (A)

b) $\{1\} \setminus \{2\} = \emptyset$ (F)

c) $\{5\} \cup \{1\} = \{1, 5\}$ (A)

d) $\{5\} \cap \{1, 5\} = \emptyset$ (F)

e) $\{1, 5\} \cup \emptyset = \{1, 5\}$ (F)

f) $\{2, 3, 4\} \cap \emptyset = \emptyset$ (F)

g) $\{2, 3\} \setminus \mathbb{N} = \emptyset$ (A)

$$3. A = \{1, 2, 3, 5\}$$

$$B = \{0, 2, 4, 6\}$$

$$C = \{1, 3, 6, 7, 8, 9\}$$

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$$

$$B \cup A = \{0, 1, 2, 3, 4, 5, 6\}$$

$$A \cap B = \{2\}$$

$$(A \cup B) \cup C = \{0, 1, 2, 3, 4, 5, 6\} \cup \{1, 3, 6, 7, 8, 9\} = \\ = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A \cup (B \cup C) = \{1, 2, 3, 5\} \cup \{0, 1, 2, 3, 4, 6, 7, 8, 9\} = \\ = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B \cap A = \{2\}$$

$$A \setminus B = \{1, 3, 5\}$$

$$B \setminus A = \{0, 4, 6\}$$

$$(A \setminus B) \setminus C = \{1, 3, 5\} \setminus \{1, 3, 6, 7, 8, 9\} = \{5\}$$

$$A \setminus (B \setminus C) = \{1, 2, 3, 5\} \setminus \{0, 2, 4\} = \{1, 3, 5\}$$

$$4. A = \{x \in \mathbb{N}^* \mid x \leq 16\} = \{1, 2, 3, 4, 5, 6, \dots, 15, 16\}$$

$$B = \{x \in \mathbb{N}^* \mid x^3 \leq 64\} = \{1, 2, 3, 4\}$$

$$x^3 \leq 64 \Leftrightarrow x^3 \leq 4^3 \Rightarrow x \leq 4 \mid \Rightarrow x \in \{1, 2, 3, 4\}$$

$x \in \mathbb{N}^*$

$$A \cap B = \{1, 2, 3, 4\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, \dots, 15, 16\}$$

$$A \setminus B = \{5, 6, 7, \dots, 15, 16\}$$

$$B \setminus A = \emptyset$$

$$5. C = \{a, b, c\}$$

$$a) \begin{array}{l} A \cup B = C \\ C = \{a, b, c\} \end{array} \mid \Rightarrow \begin{array}{l} A = \{a, b\} \\ B = \{c\} \end{array} \quad \text{atau} \quad \begin{array}{l} A = \{a\} \\ B = \{b, c\} \end{array} \dots$$

$$b) \begin{array}{l} A \cap B = C \\ C = \{a, b, c\} \end{array} \mid \Rightarrow \begin{array}{l} A = \{a, b, c\} \\ B = \{a, b, c\} \end{array}$$

$$c) \begin{array}{l} A \setminus B = C \\ C = \{a, b, c\} \end{array} \mid \Rightarrow \begin{array}{l} A = \{a, b, c\} \\ B = \emptyset \end{array}$$

$$6. |A| = 12$$

$$|B| = 10$$

$$a) |A \cap B| = ? , \text{ dac\u0103 } |A \cup B| = 15$$

$$|A \cap B| = |A| + |B| - |A \cup B| = 12 + 10 - 15 = 22 - 15 = 7$$

$$|A \cap B| = 7$$

$$b) |A \cup B| = ? \text{ dac\u0103 } |A \cap B| = 5$$

$$|A \cup B| = |A| + |B| - |A \cap B| = 12 + 10 - 5 = 22 - 5 = 17$$

$$|A \cup B| = 17$$

$$c) |A \cup B| = |A| + |B| = 12 + 10 = 22 \quad \left(\begin{array}{l} \text{cel mai mare cardinal} \\ \text{dac\u0103 nu are niciun element comun} \end{array} \right)$$

$$|A \cup B| = |A| + |B| - |A \cap B| = 12 + 10 - 10 = 12 \quad \left(\begin{array}{l} \text{cel mai mic cardinal,} \\ \text{dac\u0103 au 10 elemente comune} \end{array} \right)$$

$$|A \cap B| = |A| + |B| - |A \cup B| = 12 + 10 - 22 = 0 \quad \left(\begin{array}{l} \text{cel mai mic cardinal} \\ \text{dac\u0103 nu are niciun element} \\ \text{comun} \end{array} \right)$$

$$|A \cap B| = |A| + |B| - |A \cup B| = 12 + 10 - 12 = 10 \quad \left(\begin{array}{l} \text{cel mai mare cardinal} \\ \text{dac\u0103 au 10 elemente comune} \end{array} \right)$$

$$7. A = \{1, 2, x\} ; B = \{1, y, 3\}$$

$$x, y = ? ; A \cap B = A \cup B \Rightarrow A = B \Rightarrow x = 3 \text{ \u0219 } y = 2$$

8. $X, Y = ?$

a) $X \setminus Y = \{3, 4, 5\} \Rightarrow X$ contine elementele 3, 4, 5

b) $Y \cap X = \{0, 1\} \Rightarrow Y$ și X contin elementele 0, 1

c) $X \cup Y = \{0, 1, 2, 3, 4, 5\} \Rightarrow$ lui X sau lui Y apartin elementele $\{0, 1, 2, 3, 4, 5\}$

$\Rightarrow X = \{0, 1, 3, 4, 5\}$

$Y = \{0, 1, 2\}$

9. $A = \{x \in \mathbb{N} \mid 1 \leq x \leq a, a \in \mathbb{N}\}$

$B = \{x \in \mathbb{N} \mid a \leq x \leq 6, a \in \mathbb{N}\}$

$a = ?$, $a \in \mathbb{N}$ a.i. $A \cap B = \{4\}$

$A = \{1, 2, \dots, a\}$

$B = \{a, \dots, 5, 6\}$

$A \cap B = \{4\}$

$\Rightarrow a = 4$