

$$18. \quad a \in \mathbb{N} \\ a : 4 \Rightarrow a \in \mathcal{M}_4 \Rightarrow a \in \{0, 4, 8, \dots, 196, \dots, 364, \dots\} \quad (1)$$

$$a : 24 = c_1 \cdot 4 \Rightarrow a = 24 \cdot c_1 + 4 \Rightarrow a - 4 = 24 \cdot c_1$$

$$a : 36 = c_2 \cdot 4 \Rightarrow a = 36 \cdot c_2 + 4 \Rightarrow a - 4 = 36 \cdot c_2$$

$$\begin{array}{l} (a-4) : 24 \\ (a-4) : 36 \end{array} \Bigg| \Rightarrow (a-4) : [24, 36] \Rightarrow (a-4) \in \mathcal{M}_{72}$$

$$24 = 2^3 \cdot 3$$

$$36 = 2^2 \cdot 3^2$$

$$[24, 36] = 2^3 \cdot 3^2 = 8 \cdot 9 = 72$$

$$\mathcal{M}_{72} = \{0, 72, 144, 216, 288, \dots\} \Bigg| \Rightarrow (a-4) \in \{0, 72, 144, 216, \dots\} =$$

$$(a-4) \in \mathcal{M}_{72}$$

$$\Rightarrow a \in \{4, 76, 148, 220, 292, 364, 436, \dots\} \quad (2)$$

Din (1) și (2) $\Rightarrow a = 364$, deoarece îndeplinesc toate condițiile.

$$19. \quad a < 200, \quad a \in \mathbb{N}.$$

$$\begin{array}{l} a : 3 = c_1 \text{ rest } 0 \\ a : 4 = c_2 \text{ rest } 0 \\ a : 5 = c_3 \text{ rest } 0 \end{array} \Bigg| \Rightarrow \begin{array}{l} a = 3 \cdot c_1 \\ a = 4 \cdot c_2 \\ a = 5 \cdot c_3 \end{array} \Bigg| \Rightarrow \begin{array}{l} a : 3 \\ a : 4 \\ a : 5 \end{array} \Bigg| \Rightarrow$$

$$\Rightarrow a : [3, 4, 5] \Rightarrow a : 60 \Rightarrow a \in \mathcal{M}_{60} = \{0, 60, 120, 180, \dots\} \quad (1)$$

$$a : 7 = c_4 \text{ rest } 1 \Rightarrow a = 7 \cdot c_4 + 1 \Rightarrow a - 1 = 7 \cdot c_4 \Rightarrow (a-1) : 7 \Rightarrow$$

$$\Rightarrow (a-1) \in \mathcal{M}_7 = \{0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, \dots, 119, 126\} \Rightarrow$$

$$\Rightarrow a \in \{1, 8, 15, 22, \dots, 120, 127, \dots\} \quad (2)$$

Din (1) și (2) $\Rightarrow a = 120$, deoarece îndeplinesc toate condițiile.

20. $20 < a < 30$, $a \in \mathbb{N}$, $a = \text{nr. de elevi}$

$$\begin{aligned} a : 2 = c_1 &\Rightarrow a = c_1 \cdot 2 \\ a : 3 = c_2 &\Rightarrow a = c_2 \cdot 3 \end{aligned} \quad \Bigg| \Rightarrow \begin{aligned} a : 2 & \\ a : 3 & \end{aligned} \Bigg| \Rightarrow a : [2, 3] \Rightarrow$$

$c_1 =$ reprezintă nr. de bănci ocupate cu câte 2 copii

$c_2 =$ reprezintă nr. de bănci ocupate cu câte 3 copii

$$\Rightarrow a : 6 \Rightarrow a \in \mathcal{M}_6 = \{0, 6, 12, 18, 24, 30, \dots\} \Bigg| \Rightarrow a = 24 \text{ elevi}$$

$20 < a < 30$.

* 24 elevi, dacă rămâne o bancă neocupată (liberă)

21. a) $a : 6 = c_1$ rest 5 $\Rightarrow a = 6 \cdot c_1 + 5 \quad | +1 \Rightarrow a+1 = 6 \cdot c_1 + 6$
 $a : 5 = c_2$ rest 4 $\Rightarrow a = 5 \cdot c_2 + 4 \quad | +1 \Rightarrow a+1 = 5 \cdot c_2 + 5 \quad | \Rightarrow$

$$\Rightarrow \begin{aligned} a+1 = 6 \cdot (c_1+1) \\ a+1 = 5 \cdot (c_2+1) \end{aligned} \quad \Bigg| \Rightarrow \begin{aligned} (a+1) : 6 \\ (a+1) : 5 \end{aligned} \quad \Bigg| \Rightarrow (a+1) : [5, 6] \Rightarrow$$

$$\Rightarrow (a+1) : 30 \Rightarrow (a+1) \in \mathcal{M}_{30} = \{0, 30, 60, 90, \dots\} \Bigg| \Rightarrow$$

$a \in \mathbb{N}$

$$\Rightarrow a \in \{29, 59, 89, \dots\}$$

$a = 29$ este cel mai mic nr. natural care îndeplinește condițiile.

b) \overline{atrc}

$$\overline{atrc} : 72 = c_1 \text{ rest } 69 \Rightarrow \overline{atrc} = 72 \cdot c_1 + 69 \quad | +3 \Rightarrow$$

$$\overline{atrc} : 60 = c_2 \text{ rest } 57 \Rightarrow \overline{atrc} = 60 \cdot c_2 + 57 \quad | +3$$

$$\Rightarrow \begin{aligned} \overline{atrc} + 3 = 72 \cdot c_1 + 72 = 72 \cdot (c_1 + 1) \\ \overline{atrc} + 3 = 60 \cdot c_2 + 60 = 60 \cdot (c_2 + 1) \end{aligned} \quad \Bigg| \Rightarrow \begin{aligned} (\overline{atrc} + 3) : 72 \\ (\overline{atrc} + 3) : 60 \end{aligned} \Bigg| \Rightarrow$$

$$\Rightarrow (\overline{abc} + 3) : [60, 72] \Rightarrow (\overline{abc} + 3) : 360 \Rightarrow$$

$$60 = 2^2 \cdot 3 \cdot 5$$

$$72 = 2^3 \cdot 3^2$$

$$[60, 72] = 2^3 \cdot 3^2 \cdot 5 = 8 \cdot 9 \cdot 5 = 360$$

$$\Rightarrow (\overline{abc} + 3) \in \mathcal{M}_{360} = \{0, 360, 720, 1080, \dots\} \mid -3 \Rightarrow$$

$$\Rightarrow \overline{abc} \in \{357, 717\}$$

c) $\overline{abc} = ?$

$$\overline{abc} : 15 = c_1 \text{ h } 10 \Rightarrow \overline{abc} = c_1 \cdot 15 + 10 \quad | +5$$

$$\overline{abc} : 12 = c_2 \text{ h } 7 \Rightarrow \overline{abc} = c_2 \cdot 12 + 7 \quad | +5 \Rightarrow$$

$$\overline{abc} : 80 = c_3 \text{ h } 75 \Rightarrow \overline{abc} = c_3 \cdot 80 + 75 \quad | +5$$

$$\Rightarrow \begin{array}{l} \overline{abc} + 5 = c_1 \cdot 15 + 15 \\ \overline{abc} + 5 = c_2 \cdot 12 + 12 \\ \overline{abc} + 5 = c_3 \cdot 80 + 80 \end{array} \left| \Rightarrow \begin{array}{l} \overline{abc} + 5 = 15 \cdot (c_1 + 1) \\ \overline{abc} + 5 = 12 \cdot (c_2 + 1) \\ \overline{abc} + 5 = 80 \cdot (c_3 + 1) \end{array} \right| \Rightarrow$$

$$\Rightarrow (\overline{abc} + 5) : 15 \left| \right.$$

$$(\overline{abc} + 5) : 12 \left| \right.$$

$$(\overline{abc} + 5) : 80 \left| \right.$$

$$\Rightarrow (\overline{abc} + 5) : [12, 15, 80] \Rightarrow$$

$$\Rightarrow (\overline{abc} + 5) : 240 \Rightarrow (\overline{abc} + 5) \in \mathcal{M}_{240} =$$

$$= \{0, 240, 480, 720, 960, 1200, \dots\} \Rightarrow$$

$$12 = 2^2 \cdot 3$$

$$15 = 3 \cdot 5$$

$$80 = 2^4 \cdot 5$$

$$[12, 15, 80] = 2^4 \cdot 3 \cdot 5 = 3 \cdot 80 = 240$$

$$\Rightarrow \overline{abc} \in \{235, 475, 715, 955\}$$

$$22. a) \overline{atrc} = ?$$

$$\overline{atrc} : 19 = c_1 \text{ rest } 11 \Rightarrow \overline{atrc} = 19 \cdot c_1 + 11$$

$$\overline{atrc} : 13 = c_2 \text{ rest } 10 \Rightarrow \overline{atrc} = 13 \cdot c_2 + 10$$

$$\overline{atrc} \geq 100 \Rightarrow 19 \cdot c_1 + 11 \geq 100 \quad | -11 \Rightarrow 19 \cdot c_1 \geq 89 \Rightarrow$$

$$\Rightarrow c_1 \geq 5, c_1 \in \mathbb{N}.$$

$$19 \cdot c_1 + 11 = 13 \cdot c_2 + 10 \quad | -10 \Rightarrow 19 \cdot c_1 + 1 = 13 \cdot c_2 \Rightarrow$$

$$\Rightarrow (19 \cdot c_1 + 1) : 13 \Rightarrow (13c_1 + 6c_1 + 1) : 13 \Rightarrow 13c_1 : 13 \text{ și } (6c_1 + 1) : 13 \Rightarrow$$

$$\Rightarrow (6c_1 + 1) \in \mathcal{M}_{13} = \{0, 13, 26, 39, 52, 65, 78, 91, \dots\}$$

$$6c_1 + 1 = 0 \Rightarrow \text{nu convine}$$

$$6c_1 + 1 = 13 \Rightarrow 6c_1 = 13 - 1 \Rightarrow 6c_1 = 12 \Rightarrow c_1 = 2 \text{ nu convine}$$

$$6c_1 + 1 = 26 \Rightarrow 6c_1 = 26 - 1 \Rightarrow 6c_1 = 25 \text{ (nu convine, } c_1 \notin \mathbb{N} \text{ deoarece } c_1 \geq 5)$$

$$6c_1 + 1 = 39 \Rightarrow 6c_1 = 39 - 1 \Rightarrow 6c_1 = 38 \text{ (nu convine, } c_1 \notin \mathbb{N})$$

$$6c_1 + 1 = 52 \Rightarrow 6c_1 = 52 - 1 \Rightarrow 6c_1 = 51 \text{ (---, } c_1 \notin \mathbb{N})$$

$$6c_1 + 1 = 91 \Rightarrow 6c_1 = 91 - 1 \Rightarrow 6c_1 = 90 \Rightarrow \boxed{c_1 = 15} \in \mathbb{N}.$$

$$\overline{atrc} = 19 \cdot 15 + 11 = 285 + 11 = 296$$

Deci, $\overline{atrc} = 296$ este cel mai mic nr. natural care îndeplinește condițiile date

b) $\overline{abc} = ?$

$\overline{abc} : 18 = c_1 \text{ rest } 13 \Rightarrow \overline{abc} = 18 \cdot c_1 + 13, c_1 \in \mathbb{N}$

$\overline{abc} : 7 = c_2 \text{ rest } 6 \Rightarrow \overline{abc} = 7 \cdot c_2 + 6, c_2 \in \mathbb{N}$

$\overline{abc} \geq 100 \Rightarrow 18 \cdot c_1 + 13 \geq 100 \quad | -13 \Rightarrow 18 \cdot c_1 \geq 87 \Rightarrow c_1 \in \mathbb{N}$

$\Rightarrow c_1 \geq 5$

$18 \cdot c_1 + 13 = 7 \cdot c_2 + 6 \quad | -6 \Rightarrow 18 \cdot c_1 + 7 = 7 \cdot c_2 \Rightarrow$

$\Rightarrow (18 \cdot c_1 + 7) : 7 \Rightarrow (14c_1 + 4c_1 + 7) : 7 \Rightarrow$

$\Rightarrow [7(2c_1 + 1) + 4c_1] : 7 \Rightarrow 7(2c_1 + 1) : 7$
 $4c_1 : 7$

$4c_1 : 7 \Rightarrow 4c_1 \in d_{7,4} = \{0, 7, 14, 21, 28, \dots\}$
 $c_1 \geq 5$

$4c_1 = 0 \Rightarrow c_1 = 0$ (nu convine, $c_1 \geq 5$)

$4c_1 = 7$ nu convine $c_1 \in \mathbb{N}$.

 $4c_1 = 28 \Rightarrow c_1 = 28 : 4 \Rightarrow c_1 = 7$
 $c_1 \geq 5$

$\overline{abc} = 18 \cdot 7 + 13 = 126 + 13 = 139 \Rightarrow \overline{abc} = 139$ este cel mai mic nr. natural care indeplineste conditiile date

$$c) \overline{abcd} = ?$$

$$\overline{abcd} : 35 = c_1 \text{ rest } 29, c_1 \in \mathbb{N} \Rightarrow \overline{abcd} = 35 \cdot c_1 + 29$$

$$\overline{abcd} : 12 = c_2 \text{ rest } 7, c_2 \in \mathbb{N} \Rightarrow \overline{abcd} = 12 \cdot c_2 + 7$$

$$\overline{abcd} \geq 1000 \Rightarrow 35 \cdot c_1 + 29 \geq 1000 \quad | -29 \Rightarrow 35 \cdot c_1 \geq 971 \Rightarrow$$

$$\Rightarrow c_1 \geq 971 : 35 \Rightarrow c_1 \geq 27,74 \quad \left. \begin{array}{l} c_1 \in \mathbb{N} \end{array} \right\} \Rightarrow c_1 \geq 28$$

$$35 \cdot c_1 + 29 = 12 \cdot c_2 + 7 \quad | -7 \Rightarrow 35 \cdot c_1 + 22 = 12 \cdot c_2 \Rightarrow$$

$$\Rightarrow (35 \cdot c_1 + 22) : 12 \Rightarrow (24c_1 + 11c_1 + 12 + 10) : 12 \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} (24c_1 + 12) : 12 \\ (11c_1 + 10) : 12 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} 12 \cdot (2c_1 + 1) : 12 \\ (11c_1 + 10) : 12 \end{array} \right.$$

$$(11c_1 + 10) : 12 \Rightarrow (11c_1 + 10) \in \mathcal{M}_{12} = \{0, 12, 24, \dots, 384, 396, \dots\} \Rightarrow c_1 \geq 28$$

$$\Rightarrow 11c_1 + 10 = 384 \quad | -10 \Rightarrow 11c_1 = 374 \Rightarrow c_1 = 374 : 11 \Rightarrow \boxed{c_1 = 34}$$

$$\overline{abcd} = 35 \cdot 34 + 29 = 1190 + 29 = 1219 \Rightarrow \boxed{\overline{abcd} = 1219} \text{ cel}$$

mai mic nr. natural care indeplineste conditiile date